MATH 4400 SAMPLE EXAM

1) Find all solutions of $x^{34} = 9$ in $\mathbb{Z}/17\mathbb{Z}$.

2)
  a) Use the Euclidean algorithm to calculate d, the g.c.d. of 14 and 46.
  b) Then find integers x and y such that $46x + 14y = d$.

3) Use that $10 \equiv -1 \pmod{11}$ to show that an integer is divisible by 11 if an only if the alternating sum of its digits is divisible by 11. For example $9482 = 11 \cdot 862$, and the alternating sum of its digits is $9 - 4 + 8 - 2 = 11$.

4) The proof of the theorem of Lagrange goes as follows: Let G be a finite group and g an element in G. Let k be the order of g. Then elements of the group G can be arranged into a rectangle with rows $x, xg, xg^2 \ldots xg^{k-1}$ where x is in G. Illustrate this with $G = (\mathbf{Z}/13\mathbf{Z})^\times$ and $g = 3$.

5) Let $S = \{p_1, \ldots, p_n\}$ be a set of primes. Set $m = 4p_1 \cdots p_n + 3$. Show that m is divisible by a prime $q \equiv 3 \pmod{4}$ which is not in the set S. Conclude that there are infinitely many primes congruent to 3 modulo 4.

6) Solve the system of congruences

$$x \equiv 11 \pmod{19}$$
$$x \equiv 16 \pmod{31}$$

7) Let G be a group and e the identity element in G. Assume that $a^2 = e$ for every a in G. Show that G is commutative, i.e. $ab = ba$ for all a and b in G. .