SAMPLE EXAM I, MATH 2270-1

1) Compute, if they are defined, A^2, AB and BA for the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 3 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

2) Find the values of h for which the vector $v = \begin{bmatrix} h \\ 2 \\ -3 \end{bmatrix}$ is a linear combination (i.e. lies in the span) of

$$v_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

3) Write $v = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(v_1) = e_1$ and $T(v_2) = e_1$, where e_1 and e_2 are the columns of the 2×2 identity matrix. What is $T(v)$?

4) Find all solutions $x \in \mathbb{R}^4$ of the homogeneous system $Ax = 0$ where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 4 \\ 3 & 3 & 7 & 9 \end{bmatrix}.$$

State how many free variables there are. Let $v_1, v_2, v_3.v_4 \in \mathbb{R}^3$ be the columns of the matrix A. Prove that they are linearly dependent.

5) Generalizing the previous exercise, prove that any four vectors $v_1, v_2, v_3.v_4$ in $\mathbb{R}^3$ are linearly dependent.