MATH 2210 - FINAL EXAM PROBLEMS

1) The parametrized curve $\sigma(t) = (1, t^3, t)$ intersects the the surface $x^2 + z = 2$ in a point P.

a) Find coordinates of the point P.
b) Write a parametric equation of the line tangent to $\sigma(t)$ at the point P.
c) Write an equation of the tangent plane to the surface at the point P.

2) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}.$$

3)
a) Sketch the region of integration for the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{y}\cos(x^4)dxdy.$$

b) Compute the integral (Hint: reverse the order of integration).

4) Find the minimum and the maximum value of the function $f(x, y) = 4xy$ along the ellipse $x^2 + 4y^2 = 4$.

5) Find and analyze the critical points of the function $f(x, y) = 2x^4 - x^2 + 3y^2$.

6) Use a linear change of variables to evaluate the integral

$$\int_S xy\ dxdx$$

where S is a parallelogram with vertices $(0, 0)$, $(2, 1)$, $(1, 2)$ and $(3, 3)$.

7) Evaluate the line integral by finding a potential function.

$$\int_{(1,1)}^{(2,2)} (3x^2 + 2xy^2)dx + (3y^2 + 2x^2y)dy.$$

8) Let C be the closed curve obtained by going from $(0, 0)$ to $(1, -1)$ along $y = -x$, then going to $(1, 1)$ along $x = 1$, and then coming back to $(0, 0)$ along $y = x$. Calculate

$$\int_C xdy$$

in two ways:
a) Directly.
b) Using the Green's theorem.

9) Consider the vector field $\mathbb{V} = xi + yj$. Calculate the flux of $\mathbb{V}$ accross the boundary of the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$ in two ways:
a) Directly.
b) Using the Divergence theorem.

10) Consider the vector field $\mathbb{V}(x,y) = -yi + xj$. Let $\mathbb{V}_\varphi$ be the vector field obtained by rotating $\mathbb{V}$ counter-clockwise for the angle φ. Find the flux of $\mathbb{V}_\varphi$ accros the circle $x^2 + y^2 = 1$ two ways:
a) Directly.
b) Using the divergence theorem.
Hint: You might want to do this for $\mathbb{V}$ first. In general, for the second part, you will need to calculate $\mathbb{V}_\varphi$ explicitely. That isn't too difficult! You just need to rotate the unit vectors i and j for the angle φ.