

SAMPLE EXAM I, MATH 4030

1) Use mathematical induction to prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

2) Define inductively the function $f(n) = n^2$, where n is the natural number.

3) Write down axioms of an abelian group G . Use $+$ to denote the binary operation and 0 to denote the identity element. Using group axioms show that $a + b = a + c$ implies $b = c$.

4) Find a continued fraction expansion of $1 + \sqrt{3}$.

5) Express $0.12\overline{345}$ as a fraction.

6) Let \sim be a relation on the set of natural numbers where $x \sim y$ if $x + y$ is an even integer. Show that \sim is an equivalence relation. Which numbers are contained in $[1]$, the class of 1, and $[2]$, the class of 2. (Recall that the class of x consist of all integers y related to x .)