1) Let $V$ be a normed space and $U$ a closed subspace. For every $x \in V$, let $||x + U|| = \inf\{||x + y|| \mid y \in U\}$.

   (1) Prove that $||x + U||$ is a norm on $V/U$.
   (2) If $V$ is complete, so is $V/U$.
   (3) Let $P : V \to V/U$ be $P(x) = x + U$. Prove that $P$ has norm 1.

2) Let $T : V \to W$ be a bounded map. Let $U$ be the kernel of $T$.

   (1) Prove that $U$ is closed.
   (2) Prove that $||S|| = ||T||$ where $S : V/U \to W$ such that $S(x + U) = T(x)$, for all $x \in V$.

3) Let $T : V \to W$ be a map between two normed spaces, where $W$ is finite dimensional. Let $U$ be the kernel of $T$. Prove that $T$ is bounded if $U$ is closed.

4) Let $(X, \mathcal{M}, \mu)$ be a measure space, and $E \in \mathcal{M}$ of finite and positive measure. Let $T : L^1(X) \to \mathbb{R}$ be defined by $T(f) = \int_E f$. Prove that $T$ is a bounded functional, and compute its norm.

5) Let $T : V \to U$ be a bounded map between two normed spaces. Let $T^* : U^* \to V^*$ be defined by $T^*(f) = f \circ T$ for all $f \in U^*$ (the adjoint map).

   (1) Prove that $||T^*|| = ||T||$.
   (2) If we identify $V$ and $U$ with their canonical images in $V^{**}$ and $U^{**}$ prove that the restriction of $T^{**}$ to $V$ coincides with $T$.

6) (More magic.) Let $V$ be a space complete with respect to two norms $||\cdot||_1 \leq ||\cdot||_2$. Prove that the norms are equivalent.

7) Let $T : V \to U$ be a linear map between two Banach spaces such that if $f$ is a continuous functional on $U$, then $f \circ T$ is a continuous functional on $V$. Prove that $T$ is bounded. Hint: prove that the graph of $T$ is closed, to that end use Theorem 5.8 c on page 159.