1) In this and the following problem, use $d(x, y) = \max(|x_1 - y_1|, |x_2 - y_2|)$ as the distance function on $\mathbb{R}^2$. Use $\epsilon$-$\delta$ definition of continuity to prove that the multiplication map $\mathbb{R}^2 \to \mathbb{R}$ is continuous.

2) Let $p_i : \mathbb{R}^2 \to \mathbb{R}$ be the projection on the $i$-th coordinate. Prove that $p_i$ is continuous. Let $(X, d)$ be a metric space. Let $f : X \to \mathbb{R}^2$ be a map, and write $f(x) = (f_1(x), f_2(x))$ for every $x \in X$. In particular we have two functions $f_i : X \to \mathbb{R}$, $i = 1, 2$. Prove that $f$ is continuous if and only if $f_1$ and $f_2$ are.

3) Let $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^n$. Use the inductive definition $x^n = x \cdot x^{n-1}$ and previous exercises to prove that $f$ is continuous.

4) Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that $f(x) \geq 0$ for all $x \in [a, b]$. Prove that $\int_a^b f = 0$ implies $f(x) = 0$ for all $x \in [a, b]$. 