1) Let \( f : [a, b] \to \mathbb{R} \) be a continuous function such that \( f \geq 0 \). Show that \( f = 0 \) if and only if \( \int_a^b f(x) \, dx = 0 \).

2) Let \( V = C[a, b] \) the space of continuous real valued functions on a closed segment \([a, b]\). Prove that

\[
(f, g) = \int_a^b f(x)g(x) \, dx
\]

defines a dot product on \( V \).

3) Let \( f : [0, 1] \to \mathbb{R} \) be defined by \( f(x) = 1 \) if \( x \) is in the Cantor set and \( f(x) = 0 \) otherwise. Prove that \( f \) is Riemann integrable and compute its integral.

4) Let \( f : [1, \infty) \to \mathbb{R} \) be a non-negative function such that \( f \) is integrable on the closed segment \([1, c]\) for every \( c \geq 1 \). One can define \( \int_1^\infty f(x) \, dx \) as the supremum of \( \int_1^c f(x) \, dx \) over all \( c \). Assume that \( f \) is monotone decreasing. Prove that \( \int_1^\infty f(x) \, dx \) is finite if and only if \( \sum_{n=1}^\infty f(n) \) converges. Apply this to prove that the series \( \sum_{n=1}^\infty n^{-s} \) for \( s > 0 \) is convergent if and only if \( s > 1 \).