MATH 3210-4, HW II
DUE MONDAY SEP 24

1) A complex number z is called algebraic if there exists integers $a_0, a_1, \ldots, a_n$, such that $a_n z^n + \cdots + a_1 z + a_0 = 0$. Prove that the set of algebraic numbers is countable.

2) Prove that the following two (X, d) are metric spaces:

- $X = \mathbb{R}^2$ and $d((x_1, x_2), (y_1, y_2)) = \max(|x_1 - y_1|, |x_2 - y_2|)$.
- $X = \mathbb{Z}$ and $d(x, x) = 0$ or $d(x, y) = \frac{1}{2^n}$, if $x \neq y$, where 2^n is the largest power of 2 dividing $x - y$.

3) Let (X, d) be a metric space. The "closed" ball centered at x and of radius $r > 0$ is the set of $y \in X$ such that $d(x, y) \leq r$. Prove that the complement of the closed ball is an open set in X.

4) Let (X, d) be a metric space. Let $E \subset X$. A point $x \in E$ is interior if there exists $\epsilon > 0$ such that $B(x, \epsilon) \subset E$. Let E° be the set of all interior points of E. Prove that E° is an open set. If $O \subset E$ is an open set, prove that $O \subset E^\circ$.

5) Let $K \subset \mathbb{R}$ be a set consisting of 0 and all $1/n$, $n = 1, 2, 3, \ldots$. Prove that K is compact directly using the definition, i.e. every open cover has a finite subcover.

6) Prove that convex sets in $\mathbb{R}^2$ are connected, in the sense of definition 2.45. (The proof is very short if you do it the right way.)