MATH 4030, SECOND HW SOLUTIONS

2-1 a) If z and y are identity elements then $z + y = z$ since y is identity, but also $z + y = y$ since z is identity. Thus $z = y$.

This shows that there is only one identity element which is customarily denoted by 0 since we have chosen + to denote the operation. This 0 is not THE real number 0.

b) If t and u are two inverses of s then

$$(t + s) + u = 0 + u = u$$

on one hand, and on the other hand

$$t + (s + u) = t + 0 = t.$$

Thus $t = s$ by associativity.

This shows that the inverse unique and it is customarily DENOTED by $-s$. $-$ in front stands for “the inverse of s”.

c) Let t be the inverse of s. The equation $s + t = 0$ clearly shows that s is also the inverse of t. Thus s is the inverse of the inverse of s.

2-2 a)

$$0 + 0 = 0$$
$$s \times (0 + 0) = s \times 0$$
$$s \times 0 + s \times 0 = s \times 0$$

by distributive property. Subtract $s \times 0$ from both sides to get $s \times 0 = 0$.

b)

$$t + (-t) = 0$$
$$s \times (t + (-t)) = s \times 0$$

Now by part a) we know that $s \times 0 = 0$, thus

$$s \times t + s \times (-t) = 0$$

and this says that $s \times (-t)$ is the inverse of $s \times t$ i.e $s \times (-t) = -s \times t$.

c) This is just the iteration of part b) and using 2-1 part c).

$$(-s) \times (-t) = -(s \times (-t)) = -(-(s \times t)) = s \times t.$$

2-3 This problem is identical to 2-1. Just replace + by $\times$, 0 by 1 and $-s$ by $1/s$.

2-4 a) If a and b are positive integers i.e. natural numbers then $a \times b$ is a natural numbers so non-zero. If a is negative and b positive then $a \times b = -((-a) \times b)$ so it is negative, non-zero etc. b) $ab = cb$ implies $(a - c)b = 0$, thus $a - c = 0$ by a).