MATH 4400 SAMPLE FINAL EXAM

1) Compute 3^{25} modulo 45. Hint: compute 3^{25} modulo 9 and 5, then use CRT.

2) Solve the congruence

$$x^5 \equiv 3 \pmod{64}.$$

3) Let g be a group element such that $g^9 = e$ and $g^{16} = e$ where e is the identity element. Show that $g = e$.

4) Let R be a ring. Let 0 and 1 denote the identity elements for addition and multiplication, respectively. For every $r \in R$ prove that

- $r \cdot 0 = 0$.
- $(-1) \cdot r$ is the inverse of r with respect to addition.

5) Calculate orders of all non-zero elements modulo 13.

6) State the quadratic reciprocity law. Then calculate $\left(\frac{122}{127}\right)$.

7) Let n be a positive integer. Let p be a prime divisor of n^2+3. Use the quadratic reciprocity to conclude that $p \equiv 1 \pmod{3}$. Hint: $n^2 \equiv -3 \pmod{p}$.

8) Use the previous exercise to prove that there are infinitely many primes congruent to 1 modulo 3.

9) Can Pepin's test be done with 3 replaced by 7?

10) Use the descent procedure procedure to find a solution of the equation $x^2 + y^2 = 61$ starting with $11^2 + 1^2 = 2 \cdot 61$.