ADVANCED CALCULUS

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PROBLEMS AND APPLICATIONS TO SCIENCE AND ENGINEERING

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Brandeis University

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PREFACE

During the 19th and early 20th centuries the curriculum in advanced mathematics centered around the Cours d'Analyse: the course in mathematical analysis. This three-or-more year study was a catalog of the concepts, techniques, and accomplishments of the calculus throughout mathematics and physics. During this period we saw first the emergence of differential geometry and complex analysis as separate disciplines, then the development of linear algebra, group theory, and other branches of algebra and the beginning of an intensive research into foundations of, and formulation of mathematical concepts and techniques. During this time the Cours d'Analyse fattened, as more accomplishments of calculus were added to the catalog. The mathematical research of the late 19th and early 20th centuries brought about a revolution in mathematics which not only opened up broad new areas but changed the basic approach to the subject itself. Another level of abstraction was attained, from which it became possible to scan large areas of mathematical research, observing new relations and interconnections with a more profound understanding and exposing new frontiers of discovery. What is more, it became necessary for all scientists to attain this new level. By the mid-1920's it was apparent that the Cours d'Analyse was massively unwieldy as well as out-of-date. Thus it fragmented into a collection of smaller disciplines; some remained (calculus, differential equations, differential geometry), others disappeared and were replaced by courses in more recent mathematics (point set topology, algebra, potential theory, integration theory). A piece of advanced calculus, which was important but essentially unchanged remained (series expansion, vector calculus, partial differential equations, calculus of variations). This was the course in advanced calculus. However, research in mathematics during the past forty years has been extensive in these particular subjects. The intensive and far-reaching developments in the study of differentiable manifolds and partial differential equations have cast advanced calculus in a new and important light. It was then clear that geometry and algebra form two important cornerstones for advanced calculus. In 1957, Nickerson, Spencer, and Steenrod wrote a new advanced calculus textbook which was in effect an introduction to the techniques of modern analysis. This book bore little resemblance to the existing texts in the subject, and was not successful in replacing them. However, it made the others obsolete; every text written since then must reckon with the Nickerson–Spencer–Steenrod conception of advanced calculus.

In 1963, I taught from that text to a class of exceptionally brilliant students at Princeton University. I believe that course was successful and influential for those students. As the text has no illustrative material, I developed a set of notes of "classical" advanced calculus which we used as a supplement to the text. This was the beginning of the present textbook of advanced calculus.

I began to feel that indeed algebra, geometry, and topology are cornerstones of modern mathematical analysis, but so is "classical" advanced calculus. I decided that we needed a bridge between freshman calculus and modern analysis which leaned heavily upon the techniques of algebra and the concepts of geometry. This text is an attempt at such a bridge. In 1967, I taught from a preliminary edition to a class of physics-motivated juniors, and in 1968 I taught from what is essentially the present text to a class of sophomores. These two classes have had a profound influence on the development of the text and I am deeply indebted to them for assistance in matters of style and pedagogy.

Needless to say, I did not complete the entire text in either year. As a text for juniors we covered a now-extinct chapter on metric spaces and Chapters 4–8, and in the class of sophomores we covered Chapters 1, 2, 3, 5, and 7. I feel that either of these is an adequate year's course, depending upon the structure of the preceding and subsequent courses. This text assumes only a course in calculus that includes analytic geometry, multiple integration, and partial differentiation. These topics are speedily reviewed in Chapter 2 with a view to setting up the style of the present text as well as indicating the more valuable facts and concepts of calculus. Chapter 2 includes the abstract formulation of the technique of successive approximations; this is, of course, the basic theoretical tool in advanced calculus.

Chapter 1 is hardly a course in linear algebra; it is rather a tour through those algebraic ideas and techniques which are essential to analysis. It is a large chapter and it is very likely that the student will become anxious to return to his analytic tools before the chapter is completed. It can thus be split up: Sections 1.3-1.8 are relevant to Chapters 3 and 4, because the topic of differential equations is consistently handled in the context of systems. The last four sections can be postponed until the student reaches Chapters 6–8, for it is with the geometric study of Fourier series that they begin to be relevant. Similarly, Chapter 2 can be broken up. Sections 2.6–2.9 are completely review material and can be omitted altogether if that is suitable. If the proof of Picard's theorem is omitted or delayed, this chapter is not relevant until Chapter 5. Thus Section 2.1–2.3 could be done just before beginning Chapter 5. Sections 2.4, 2.5, 2.10, and 2.11 are of a purely theoretical nature and can be kept aside until Section 3.5 is studied, or until the integral calculus in several variables is begun (Chapters 7 and 8).

Chapters 3-5 constitute a little course in ordinary differential equations. Since the study of curves and some complex variables are relevant to this topic, they are introduced in these chapters. Chapter 4, in particular, is about particle motion and Chapter 5 about series expansions in the complex domain.

Chapter 6 is devoted to the study of Fourier series and their use in the classical partial differential equations. This is the only illustration of eigenvalue expansions.

Chapters 7 and 8 form the part of advanced calculus having to do with integration; here, we find the various versions of Stokes' theorem and its applications. Outside of the notion of a differential one-form the approach is vector calculus rather than differential forms. I have included in Chapter 8 the study of geodesics and Dirichlet's principle; there is no further calculus of variations.

This text is thus intended to cover the course in advanced calculus given at the sophomore or junior level. The emphasis in the text is on concepts and techniques; my main intention being to present the *methods* of calculus. There are numerous illustrative examples and exercises on each method introduced. The exercises appear at the end of each section. Proofs of theorems are included mainly to offer further understanding of the mathematical machinery, and secondarily to illustrate its logical structure. It should be possible to read this book while skipping all proofs. The problems at the ends of the sections and the miscellaneous problems are included to deepen the student's understanding of the material, to allow him to try his hand at mathematical inference, and to suggest related topics.

I wish to thank Anne Clarke and Irene Dougherty of Brandeis University for their typing of the preliminary notes leading to this text and the classes of Mathematics 35 (1967–1968) and Mathematics 21 (1968–1969) for their assistance in correcting them. I thank also the editorial staff of W. A. Benjamin, Inc., for their patient, friendly, and expert assistance. Finally, it gives me great pleasure to thank my wife, Ricki, who not only typed the entire final manuscript and who gave me the needed encouragement to see this text through, but who also makes the world's greatest martinis.

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