MATH 5750-001: Representation Theory Techniques in Quantum Physics

TTh 10:45am-12:05pm, Room: CSC 108

General Information

Instructor: Anna Romanova, Office: JWB 115 (President's Circle), Email: `romanova@math.utah.edu`, Web Page: `www.math.utah.edu/~romanova`

Office Hours: Mondays 2-3pm, JWB 115

Weekly Problem Sessions: Wednesdays, 3-5pm, LCB 323

Textbook: Singer, Stephanie Frank. *Linearity, symmetry, and prediction in the hydrogen atom.* Springer Science & Business Media, 2006.

Course Web Page: All course information and announcements will be posted on Canvas. You can access the course's Canvas page through CIS. This page contains material that may help you succeed in this course. Check it frequently!

Prerequisites: Grade of B+ or higher in MATH 2270 (Linear Algebra), PHYS 2210 (Calculus-Based Physics I), admission by instructor consent

Course Overview: The predictive power of representation theory in quantum physics is one of the great successes of 20th century mathematics. Understanding this beautiful connection amounts to combining linear algebra with a little bit of group theory - mathematical materials readily accessible to junior- and senior-level undergraduate mathematics and physics students. This course will introduce the tools necessary to understand this interplay between mathematics and physics through a detailed exploration of the most basic example - the hydrogen atom. The course will develop all of the mathematics necessary to understand this application, starting with a review of linear algebra, and culminating with the representation theory of $SO(3)$ and $SU(2)$. Once the necessary mathematical background is established, we will use the spherical symmetry of the hydrogen atom to make predictions about its quantum states.

Course Goals and Teaching Methods

This course has two primary goals. The course is structured in a way to help the class achieve these two goals. I have constructed this course around research-based practices to best help you learn and grow in your mathematical thinking. I would like to be transparent with you about my teaching methods in order to make this course as productive for you as possible.

Goal 1: Tell a beautiful story. I find the story of how representation theory and quantum mechanics collided to be fascinating, and I believe that it is one of the best ways to motivate the study of pure mathematics. The following paragraphs come from a speech titled "What do mathematicians do?" given by Harvard professor George Mackey, and they capture exactly what I find so incredible about the relationship between representation theory and quantum mechanics.

"At this point I must make it emphatically clear that, in spite of what I have just said, pure mathematicians concern themselves very little with the external world - even in its measurable aspects. Their concern with the intellectual tools used in analyzing the external world is not so much in using these tools as in polishing them, improving them and very occasionally inventing brand new ones. Indeed it is their concern with the tools themselves, rather than with using the tools, that distinguishes them from applied mathematicians and the more mathematically minded scientists and engineers.

While it is natural to suppose that one cannot do anything very useful in tool making and tool improvement, without keeping a close eye on what the tool is to be used for, this supposition turns out to be largely wrong. *Mathematics has sort of inevitable structure which unfolds as one studies it perceptively. It is as though it were already there and one had only to uncover it. Pure mathematicians are people who have a sensitivity to this structure and such a love for the beauties it presents that they will devote themselves voluntarily and with enthusiasm to uncovering more and more of it, whenever the opportunity presents itself."*

It is well worth reading the entire text of this speech. You can find it on the website of the author of our textbook: `www.symmetrysinger.com/Mackey/speech.pdf`. The story of representation theory's role in quantum physics is one that is not usually told at the undergraduate level due to the deep mathematical machinery involved. However, I believe that by focusing on the simplest example instead of introducing the subject in its full generality, we will be able to get a glimpse of the beauty of this story without relying on a full background of graduate level mathematics. This is the main goal of this course.

Goal 2: Communicate mathematics. Communication is an important part of being a professional mathematician, physical scientist, educator, computer scientist or engineer. Mathematics is a notoriously difficult subject to communicate, which makes an ability to clearly communicate mathematics a particularly valuable skill. The secondary goal of this course is to develop our written and oral mathematics communication skills through written homework assignments typeset in Latex, in-class peer collaboration, presentation of homework problems, and a final course project. In addition to developing your scientific communication skills, this goal is intended to have the secondary effect of giving you a deeper understanding of the mathematical material. Explaining mathematics to others is one of the best ways to truly understand the material yourself.

Course Work and Evaluation

Grading: Homework, class participation, and the final project will contribute to your grade in this course. The breakdown of how each element contributes is the following:

Weekly Homework	40%
Class Participation	20%
Final Project Written Component	30%
Final Project Presentation	10%

Weekly Homework: There will be one homework assignment each week. Homework problems will be posted on canvas, and homework assignments will be due in class on Thursdays. Each student should turn in their own homework assignment, though I encourage students to work together on homework outside of class. Homework assignments should be typeset in Latex. For details on how to use Latex, see the "Latex Resources" module on Canvas. I have provided a template for the first homework assignment which includes typed problem statements. You can modify this template and use it as your starting place for all homework assignments. In all homework assignments, you should type the problem statements as I did for HW 1. I will grade a selection (not all!) of homework problems each week. I will post solutions to all homework problems on Canvas for your reference. I encourage you to use these solutions to check your own work on homework problems that were not graded.

Weekly Problem Sessions: There will be a two-hour weekly problem session where students will take turns presenting homework problems, and I will be available to help. These problem sessions will take place from **3-5pm on Wednesdays** in **LCB 323**. All students are encouraged to attend these problem sessions if their schedules allow for it, though attendance in these sessions is voluntary.

Class Participation: Class attendance is mandatory. If you have to miss class because of a legitimate excuse (e.g. illness, personal crisis, accident, etc.), please contact me as soon as possible to let me know about the situation. Your class participation score will made up from physical presence and participation in class activities.

Final Project: Instead of exams, you will complete a final course project. This project will include a 10-15 page research paper typeset in Latex and a 10-20 minute class presentation. (The precise timing of the presentation will depend on the number of groups.) Final presentations will take place on **Tuesday, May 1** from **10:30am-12:30pm** in **CSC 108**. I will suggest potential project topics throughout the semester, and a list of potential project topics can be found in Appendix C of the textbook. You are welcome to select a topic that is not on this list, but please run your idea by me before starting. This project can be done individually, or in groups of 2-3 students. If you are working in a group, you can all collaborate on the same Latex document using Overleaf. Project proposals are due on **Friday, April 6**. The last two weeks of class (the weeks of April 16th and April 23), we will dedicate class time to working on projects. The written component of the project will be due on **Friday, April 27**.

Other Policies and Resources

Math Tutoring Center: Do not hesitate to come to my office during office hours or by appointment to discuss a homework problem or any aspect of the course. The T. Benny Rushing Mathematics Tutoring Center offers free tutoring, though our class is not on the list of classes supported by the tutoring center. However, if you need help with linear algebra or spherical harmonics, you might be able to consult these tutors. Beginning the second week of classes, tutoring will be available from 8am to 8pm Monday through Thursday and 8am to 6pm on Friday. If you want to hire an outsider tutor (for a fee), you can find a list of such people through the math department.

Veteran's Center: If you are a student veteran, the University of Utah has a Veterans Support Center located in Room 161 in the Olpin Union Building. Hours: M-F 8-5pm. Please visit their website for more information about what support they offer, a list of ongoing events and links to outside resources: http://veteranscenter.utah.edu/. Please also let me know if you need any additional support in this class.

LGBT Resource Center: If you are a member of the LGBTQIA* community, I want you to know that my classroom is a safe zone. Additionally, the University of Utah has an LGBT Resource Center on campus. They are located in Room 409 in the Olpin Union Building. Hours: M-F 8-5pm. You can visit their website to find more information about the support they can offer, a list of events through the center and links to additional resources: http://lgbt.utah.edu/. Please also let me know if there is any additional support you need in this class.

The Americans with Disabilities Act: The University of Utah seeks to provide equal access to its programs, services, and activities for people with disabilities. If you will need accommodations in this class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Olpin Union Building, (801) 581-5020. CDS will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability Services.

Addressing Sexual Misconduct: Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information. If you or someone you know has been harassed or assaulted, you are encouraged to report it to the Title IX Coordinator in the Office of Equal Opportunity and Affirmative Action, 135 Park Building, 801- 581-8365, or the Office of the Dean of Students, 270 Union Building, 801-581-7066. For support and confidential consultation, contact the Center for Student Wellness, 426 SSB, 801-581-7776. To report to the police, contact the Department of Public Safety, 801-585- 2677(COPS).

Important Dates:

Drop Deadline	January 19
Withdraw Deadline	March 2
Spring Break	March 18-25
Final Project Proposal	April 6
Classes End	April 24
Final Paper Due	April 27
Project Presentations	May 1

Disclaimer: I reserve the right to change any information in this syllabus throughout the semester. If I make a change to the course policies, I will inform you in class, and post an updated version of the syllabus to canvas. I will hold you accountable for information that is stated in class or posted on canvas.