Math 2250-3 Maple Project III

Forced Oscillations and Resonance

Due November 9, 2005


Most of this project is concerned with resonance and solutions to linear second order
differential equations. In the last problem we study a similar nonlinear equation.

Problem 1. Undamped Forced Oscillations.

We take the initial value problem coming from the equation for forced undamped
oscillations

\[ x'' + 2.5x = 7 \cos(\omega t), \]
\[ x(0) = 0, x'(0) = 0. \]

a. Find the natural frequency of the system (this does not involve the \( 7 \cos(\omega t) \) term). Let \( \omega \) by five times this natural frequency and solve the resulting initial value problem using \texttt{dsolve}. Plot the solution on a suitable interval to show the global behavior of the solution.

b. The solution is a sum of two functions of different periods. Find the exact period of the solution (see Example 1 on p. 350 of the text).

Problem 2. Practical Resonance.

In this problem we consider the initial value problem

\[ x'' + cx' + 25x = 7 \cos(\omega t), \]
\[ x(0) = 0, x'(0) = 0. \]

a. Consider each of the damping constants \( c = 1/2, c = 1, \) and \( c = 2 \). Compute the amplitude function \( C(\omega) \) for these three values of \( c \) and plot the three functions on one graph. (The function \( C(\omega) \) is defined on page 355, equation (21).)

b. For each of the three values of \( c \) find the value of \( \omega \) where \( C(\omega) \) is a maximum and find the value of \( C(\omega) \) at that point. This value of \( \omega \) is the frequency of “practical resonance” in the damped case, and the amplitude \( C(\omega) \) at this frequency should become larger as \( c \) becomes smaller.

Problem 3. Resonance

a. In the equation of problem 1, replace \( \omega \) by the natural frequency of the system and solve the equation using \texttt{dsolve}. Graph the resulting solution.
b. In the equation of problem 2, graph the solution where \( c = 1/2 \) and \( \omega \) is the frequency of practical resonance for \( c = 1/2 \).

**Problem 4. A nonlinear equation.**

In this problem we consider the equation

\[
x'' + \tan x = 0
\]

starting at the equilibrium position with two different initial velocities and compare it to the linear equation

\[
x'' + x = 0
\]

with the same initial conditions. Notice that \( \tan x \) is close to \( x \) near zero but goes to infinity at \( t = \pm \pi/2 \). For this problem use DEplot to plot the solutions of the differential equations.

**a.** Plot the solution for each of these between \( t = 0 \) and \( t = 10 \) for each of the initial conditions \( x(0) = 0, x'(0) = 1 \) and \( x(0) = 0, x'(0) = 3 \) (this is four graphs in all).

**b.** Describe the difference between the behavior of the solutions to the linear and the nonlinear equations as you increase the initial velocity. Try to explain this difference by analyzing the difference between the two differential equations.

**Notes on Maple Functions.**

**Problem 1.** The format for dsolve needed here is `dsolve([ODE,ICs].x(t))`. The ODE is \( x'' + 2.5x = 7 \cos(\omega t) \), which in Maple would be written

\[
diff(x(t),t,t)+2.5*x(t)=7*cos(omega*t),
\]

where you can put in the actual value of \( \omega \) you are using. ICs are the initial conditions, in this case \( x(0)=0, D(x)(0)=0 \).

The function dsolve does not interpret the solution it gives as a function of \( t \) that can be plotted. To turn it into such a function, first define

\[
f := \text{dsolve}(...);
\]

(where you put in the above things for ...), then

\[
X := \text{unapply}(\text{rhs}(f), t);
\]

Now you can plot \( X(t) \).

**Problem 2.** In this problem you have to plot three graphs on the same axes. To graph the functions \( f(t), g(t), \) and \( h(t) \) on the same graph from \( t = a \) to \( t = b \) the command is

\[
\text{plot}([f(t), g(t), h(t)], t=a..b).
\]

Supposedly it is possible to find the maximum value by clicking at the point on the graph. Another way is to use maximize: to get the maximum value of \( f(t) \) between \( a \) and \( b \) together with the value of \( t \) where it attains the maximum use `maximize(f(t), t=a..b, location=true)`. The `location = true` is to have it return the value of \( t \). To turn the result into decimal form use `evalf`. 
Problem 4. The command `DEplot` solves the differential equation numerically. The command for the given differential equation with the first initial conditions is

\[
\text{DEplot(diff(x(t),t,t)+\tan(x(t)))=0,x(t),t=0..10,}
\]

\[
[x(0)=0,D(x)(0)=1],x=-3..3,\text{stepsize=.01});
\]

This produces a graph with \(t\) between 0 and 10 and \(x\) between \(-3\) and 3, which is a reasonable range for this example. The `stepsize=.01` is to set the length of the step small enough to give an accurate graph. It is too big in this nonlinear example the graph produced is horribly wrong— it is interesting to think about why that is the case.