Math 6320
Assignment 4
March 11, 2009

In these problems K always denotes a field.

1. Give an example where α is algebraic of degree prime to 3, but $K(\alpha) \neq K(\alpha^3)$.
2. Let E/K be a field extension of degree n. If $f(X)$ is an irreducible polynomial in $K[X]$ of degree prime to n, show that $f(x)$ is irreducible in $E[X]$.
3. If n is an odd positive integer, prove that
$$\mathbb{Q}(\cos(2\pi/n)) = \mathbb{Q}(\cos(\pi/n)) .$$
4. Determine the Galois group of $x^5 - 5p^4x + p \in \mathbb{Q}[x]$ where p is a prime integer.
5. Let G be the Galois group of $x^5 - 7 \in \mathbb{Q}[x]$. Is G solvable? Is it abelian?
6. Determine the Galois group of $x^8 - 2 \in \mathbb{Q}[x]$.
7. Let $K \subset L$ be an algebraic extension of fields. Prove that every irreducible polynomial in $L[X]$ has a nonzero multiple in $K[X]$.
8. Show that a finite field extension E of K is normal if and only if for every irreducible polynomial $f(X)$ in $K[X]$, all the irreducible factors of $f(X)$ in $E[X]$ have the same degree.