Seed Dispersal and Biological Invasion

Tom Robbins

Department of Mathematics

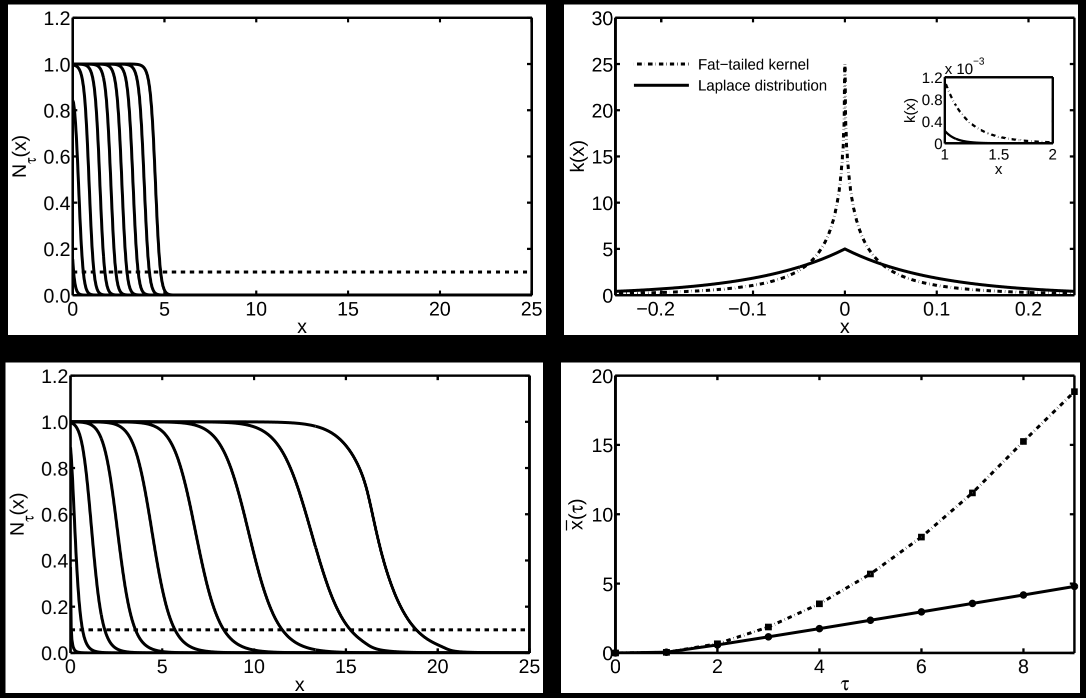
University of Utah

Outline

- Biology background
- Seed Dispersal Model
- Biological Invasions in Heterogeneous Environments

Biology background

- Population dynamics: $N_{\tau+1}(x) = \int f(N_\tau)k(x-y)dy$

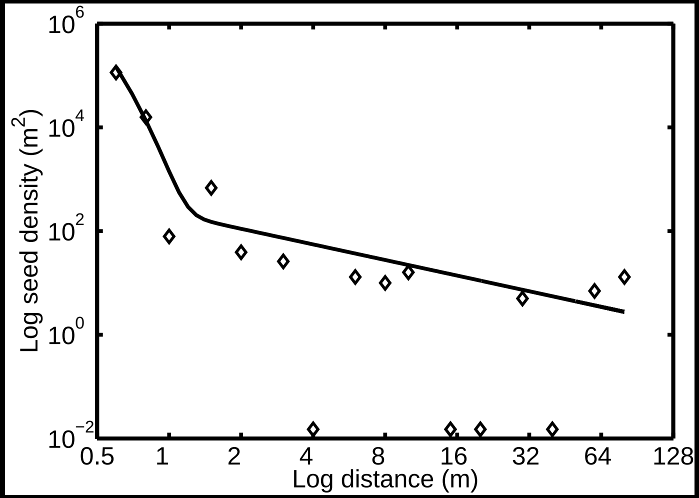


- Shape of the tail defines the rate of spread

Biology background

- Understanding biological invasions with long-distance dispersal in a heterogeneous environment
- Need for a mechanistic model to predict the shape of the tail
- A tractable model for the population dynamics that includes heterogeneity

Biology background



Bullock and Clarke [2000]

- Long-Distance dispersal events are difficult to measure
- Dispersal is not accurately described by a simple decay process
- Local and long-distance dispersal

Biology background

- Mechanistic model for seed dispersal:
  - Start from first principles
  - Turbulent transport and inertial effects
  - Estimate the tail of the dispersal kernel

Biology background

- Mechanistic model for seed dispersal:
  - Start from first principles
  - Turbulent transport and inertial effects
  - Estimate the tail of the dispersal kernel

- For a spatially heterogeneous environment, how are persistence and spread rates affected by:
  - Local growth rates
  - Seed deposition rates
  - Local vs. long-distance dispersal

Seed Dispersal Model

$$m\frac{d^2}{dt^2}\mathbf{y} = m\mathbf{G}_z + \mu_s\left(\mathbf{u}_w - \frac{d}{dt}\mathbf{y}\right)$$

- m is the seed mass
- $\mathbf{y}$ is the seed location
- $\mathbf{G}_z$ is the acceleration of gravity
- μ_s coefficient of friction for the seed
- $\mathbf{u}_w$ is the wind velocity

Seed Dispersal Model

- Inertial scaling parameter: $\epsilon = \frac{1}{T_c}\left(\frac{m}{\mu_s}\right)$ (nondimensional)

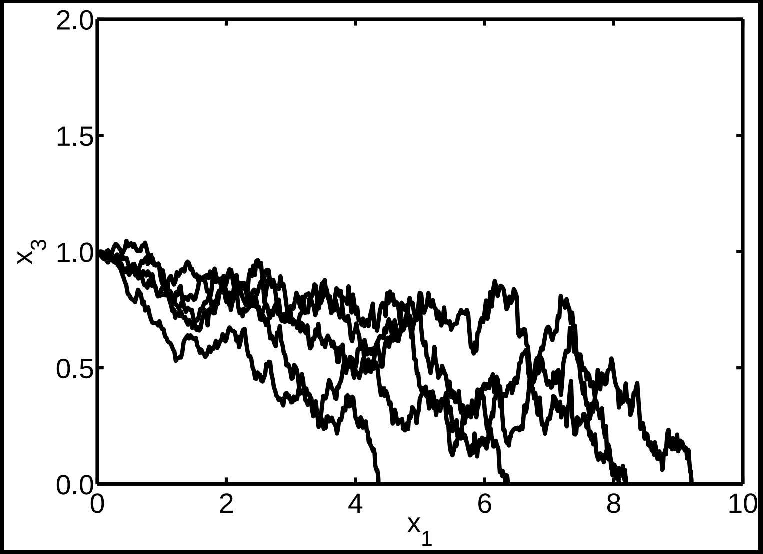
$$
\begin{aligned}
\frac{d}{dt}y_1 &= v_1 \\
\epsilon\frac{d}{dt}v_1 &= [u_w - v_1] \\
\frac{d}{dt}y_2 &= v_2 \\
\epsilon\frac{d}{dt}v_2 &= [w_w - 1 - v_2]
\end{aligned}
$$

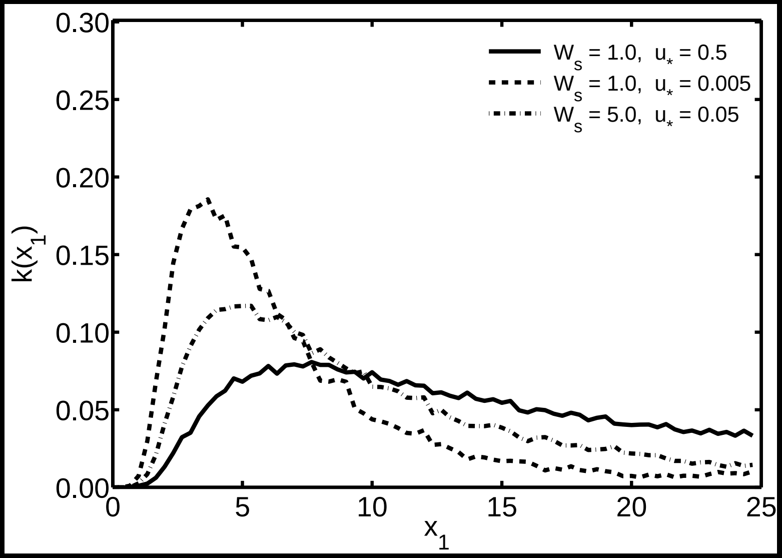
Seed Dispersal Model: SDE

$$
\begin{aligned}
dY_{1,t} &= V_{1,t}dt \\
\epsilon dV_{1,t} &= [\hat{u}(Y_2) - V_{1,t}]dt + \sqrt{2D}d\omega_t \\
dY_{2,t} &= V_{2,t}dt \\
\epsilon dV_{2,t} &= [\hat{w}(Y_2) - 1 - V_{2,t}]dt + \sqrt{2\hat{D}_z(Y_2)}d\omega_t
\end{aligned}
$$

- $(Y_{1,t}, Y_{2,t})$, $(V_{1,t}, V_{2,t})$ are random variables
- $d\omega_t$ is Brownian noise

Seed Dispersal Model: SDE





- (left) 5 realizations of the SDE model
- (right) Frequency plot of first hitting locations

Seed Dispersal Model: PDE ($\epsilon = 0$)

$$\begin{aligned}\frac{\partial \rho}{\partial t} &= D\frac{\partial^2 \rho}{\partial x_1^2} + \frac{\partial}{\partial x_2}\left\{\hat{D}_z \frac{\partial}{\partial x_2}\rho\right\} \\ &\quad -\hat{u}\frac{\partial \rho}{\partial x_1} + \frac{\partial \rho}{\partial x_2}\end{aligned}$$

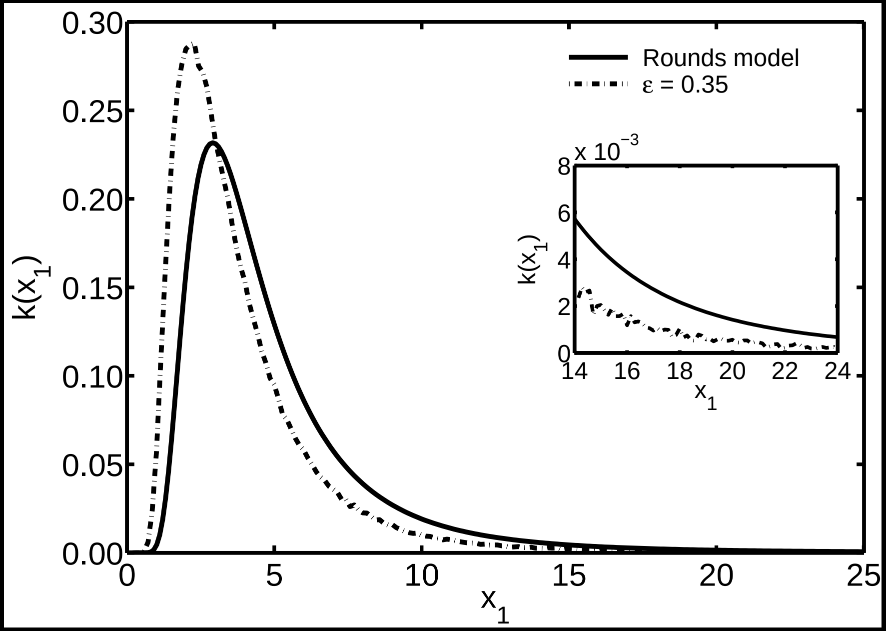
- Initial condition:

$$\rho(\mathbf{x}, 0) = \delta(x_1)\delta(x_2 - 1)$$

- Boundary condition at $x_2 = 0$:

$$\hat{D}_z \frac{\partial \rho}{\partial x_2} + \rho = V_d \rho$$

Seed Dispersal Model



- Logarithmic wind profile for $\hat{u}$
- The SDE model predicts a fast drop off in the tail
- The parabolic model predicts a “*fat tail*”

Seed Dispersal Model: PDE ($\epsilon > 0$)

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \epsilon \frac{\partial^2 \rho}{\partial t^2} \quad = \quad & D \frac{\partial^2 \rho}{\partial x_1^2} + \frac{\partial}{\partial x_2} \left\{ \hat{D}_z \frac{\partial \rho}{\partial x_2} \right\} \\
& - \hat{u} \frac{\partial \rho}{\partial x_1} + \frac{\partial \rho}{\partial x_2} \\
& + \epsilon \left[\frac{\partial^2}{\partial x_1^2} \left\{ \hat{u}^2 \rho \right\} + 2 \frac{\partial^2}{\partial x_1 \partial x_2} \left\{ \hat{u} \left(\hat{w} - 1 \right) \rho \right\} \right. \\
& \left. \qquad + \frac{\partial^2}{\partial x_2^2} \left\{ \left(\hat{w} - 1 \right)^2 \rho \right\} \right]
\end{aligned}
$$

- Limited velocity of seed propagation ($\epsilon \frac{\partial^2 \rho}{\partial t^2}$)
- Cross diffusion terms

Seed Dispersal Model

- Mechanistic basis for existing PDE models
- Derived a new, velocity limited, PDE model
- SDE model predicts a "*thin tail*"?
- PDE model predicts a "*fat tail*" ($\epsilon = 0$)
- Analysis on the new PDE model to determine the shape of the tail ($\epsilon > 0$)

Invasion Model

- Consider the invasion of terrestrial plant species

Invasion Model

- Consider the invasion of terrestrial plant species
- How is invasibility and spread rates of the population affected by:
    - local deposition of seeds
    - ratio of local to long-distance seed dispersal
    - local growth rates

Invasion Model

- Consider the invasion of terrestrial plant species
- How is invasibility and spread rates of the population affected by:
  - local deposition of seeds
  - ratio of local to long-distance seed dispersal
  - local growth rates
- Can environmental heterogeneity increase or decrease/stall an invasion?

Invasion Model

- Infinite, one-dimensional environment

Invasion Model

- Infinite, one-dimensional environment
- Growth and dispersal occur in distinct, non-overlapping stages

Invasion Model

- Infinite, one-dimensional environment
- Growth and dispersal occur in distinct, non-overlapping stages
- Integrodifference model for the population density

$$N_{\tau+1}(x) = \int_{-\infty}^{+\infty} f(N_\tau(y); y)k(x,y)dy$$

  - $N_\tau(x)$ is the population density
  - f is the nonlinear growth function
  - $k(x,y)$ dispersal kernel

Invasion Model

- Beverton-Holt growth dynamics

$$f(N;x) = \frac{r_0(x)N}{1 + [(r_0(x) - 1)N]}$$

Invasion Model

- Beverton-Holt growth dynamics

$$f(N;x) = \frac{r_0(x)N}{1 + [(r_0(x) - 1)N]}$$

- The dispersal model:

$$\begin{aligned}
\frac{\partial c_1}{\partial t} &= D_\eta \frac{\partial^2 c_1}{\partial x^2} - a(x)c_1 \\
\frac{\partial d_1}{\partial t} &= a(x)c_1 \\
c_1(x,0;y) &= w(y)\delta(x-y) \\
d_1(x,0;y) &= 0
\end{aligned}$$

  - c, d are seed densities
  - a is the deposition rate
  - w is the dispersal ratio

Invasion Model

- Beverton-Holt growth dynamics

$$f(N;x) = \frac{r_0(x)N}{1+[(r_0(x)-1)N]}$$

- The dispersal model:

$$\begin{aligned}
\frac{\partial c_1}{\partial t} &= D_\eta \frac{\partial^2 c_1}{\partial x^2} - a(x)c_1 \\
\frac{\partial d_1}{\partial t} &= a(x)c_1 \\
c_1(x,0;y) &= w(y)\delta(x-y) \\
d_1(x,0;y) &= 0
\end{aligned}
\qquad
\begin{aligned}
\frac{\partial c_2}{\partial t} &= D \frac{\partial^2 c_2}{\partial x^2} - a(x)c_2 \\
\frac{\partial d_2}{\partial t} &= a(x)c_2 \\
c_2(x,0;y) &= (1-w(y))\delta(x-y) \\
d_2(x,0;y) &= 0
\end{aligned}$$

  - c, d are seed densities
  - a is the deposition rate
  - w is the dispersal ratio

Invasion Model

- Local and long-distance dispersal scales: $D_\eta \ll D$
- The dispersal kernel is the steady-state seed concentration on the ground:

$$
\begin{aligned}
k(x,y) &= k_1(x,y;D_\eta) + k_2(x,y) \\
&= \lim_{t\to\infty} \{d_1(x,t;y) + d_2(x,t;y)\}
\end{aligned}
$$

- The IDE model is

$$
\begin{aligned}
N_{\tau+1}(x) &= \int_{-\infty}^{+\infty} f(N_\tau(y);y)k_1(x,y;D_\eta)dy \\
&+ \int_{-\infty}^{+\infty} f(N_\tau(y);y)k_2(x,y)dy
\end{aligned}
$$

Invasion Model

- Periodically divide the environment into *good* ($r_0 > 1$) and *bad* ($r_0 < 1$) patches:

$$r_0(x) = \begin{cases} r_1 & \text{if } 0 \leq x < x_a \\ r_2 & \text{if } x_a \leq x < l\,; \qquad r_0(x+l) = r_0(x) \end{cases}$$

Invasion Model

- Periodically divide the environment into *good* ($r_0 > 1$) and *bad* ($r_0 < 1$) patches:

$$r_0(x) = \begin{cases} r_1 & \text{if } 0 \leq x < x_a \\ r_2 & \text{if } x_a \leq x < l\,; \qquad r_0(x+l) = r_0(x) \end{cases}$$

- Divide the environment into *high* ($a = 1$) and *low* ($a < 1$) deposition patches:

$$a(x) = \begin{cases} a_1 & \text{if } 0 \leq x < x_a \\ a_2 & \text{if } x_a \leq x < l\,; \qquad a(x+l) = a(x) \end{cases}$$

- Local ($w = 1$) and long-distance ($w < 1$) dispersal:

$$w(x) = \begin{cases} w_1 & \text{if } 0 \leq x < x_a \\ w_2 & \text{if } x_a \leq x < l\,; \qquad w(x+l) = w(x) \end{cases}$$

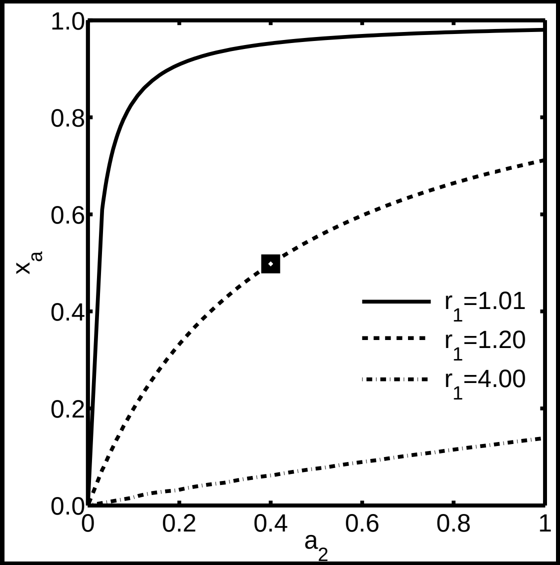
Invasion Model: Colony Persistence

- A colony is said to persist in an environment if when initially introduced at low densities, the population density eventually increases

Invasion Model: Colony Persistence

- A colony is said to persist in an environment if when initially introduced at low densities, the population density eventually increases

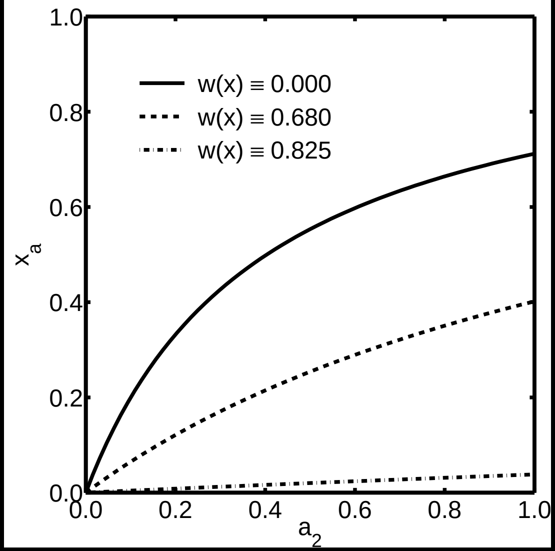
- $w(x) = 0$ and *high* deposition in *good* patches
- Deposition rate in *bad* patch (a_2) vs. relative fraction of *good* patches (x_a)



- Region of persistence is located above the curve

Invasion Model: Colony Persistence

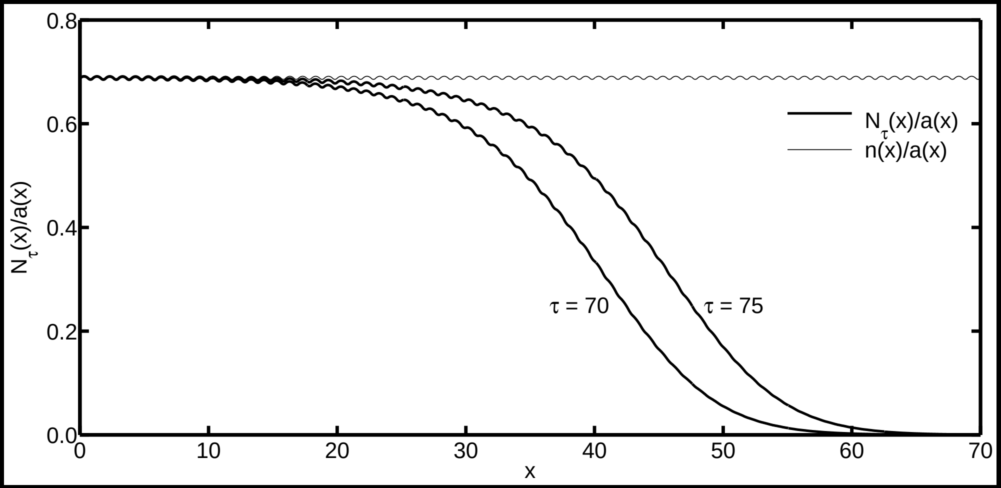
- $0 \leq w(x) \leq 1$ and *high* deposition in *good* patches
- Deposition rate in *bad* patch (a_2) vs. relative fraction of *good* patches (x_a)



- Local dispersal increases species persistence

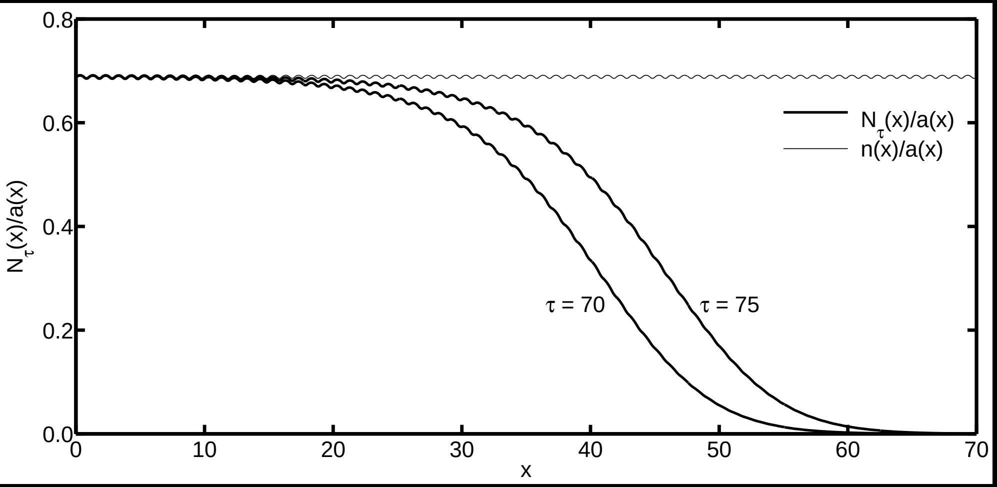
Invasion Model: Traveling Periodic Wave

- For a persistent species: $N_\tau(x)$ for τ large



Invasion Model: Traveling Periodic Wave

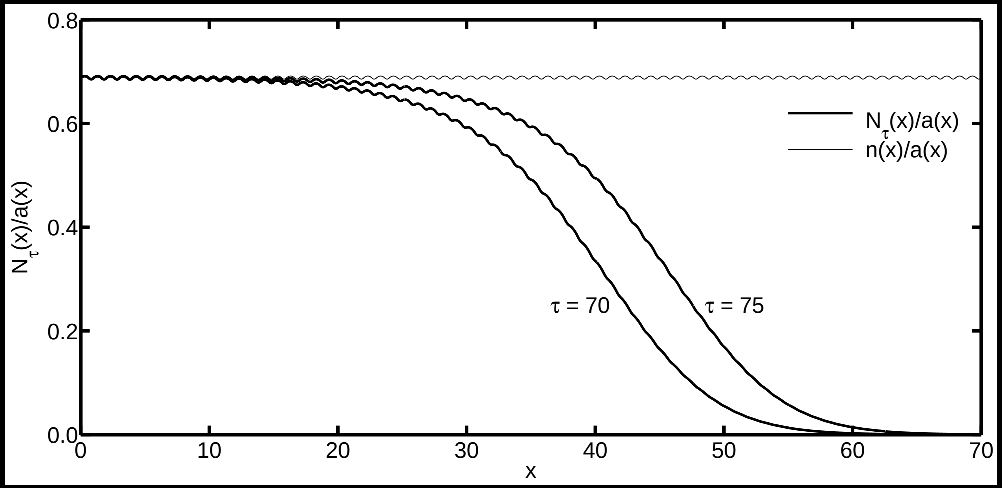
- For a persistent species: $N_\tau(x)$ for τ large



- Dispersion relation for the speed of the wave

Invasion Model: Traveling Periodic Wave

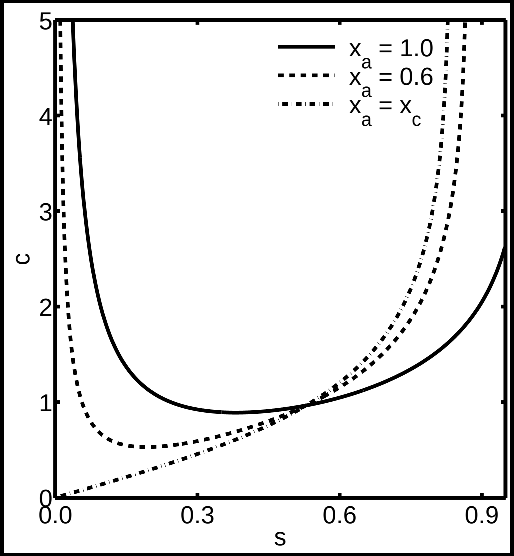
- For a persistent species: $N_\tau(x)$ for τ large



- Dispersion relation for the speed of the wave
- How does heterogeneity affect the wave speed?

Invasion Model: Traveling Periodic Wave

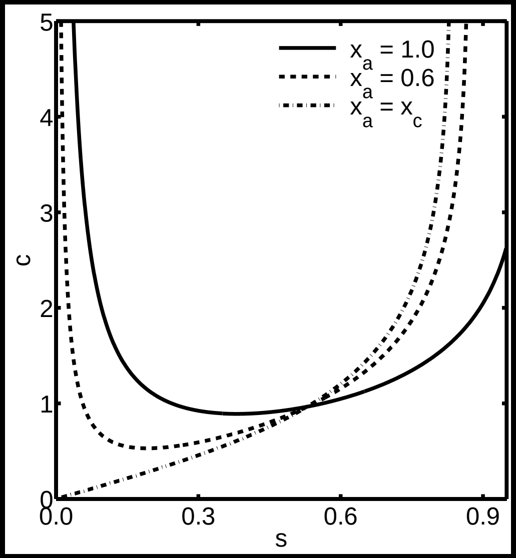
- $w(x) = 0$ and *high* deposition in *good* patches
- Dispersion relation for the speed of the wave:

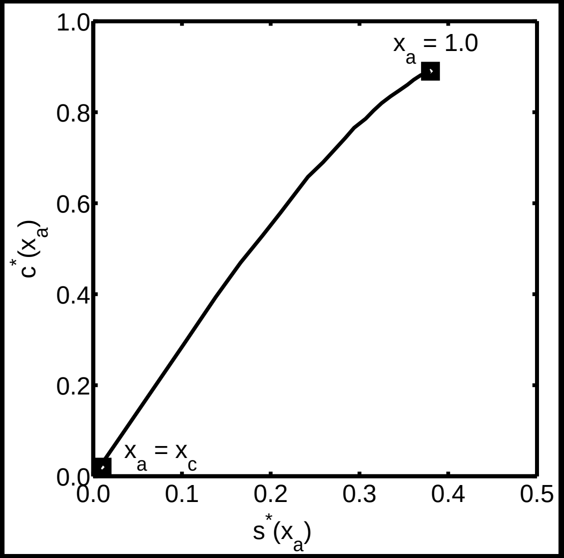


- The rate of spread is the minimum $c(s)$

Invasion Model: Traveling Periodic Wave

- $w(x) = 0$ and *high* deposition in *good* patches
- Dispersion relation for the speed of the wave:

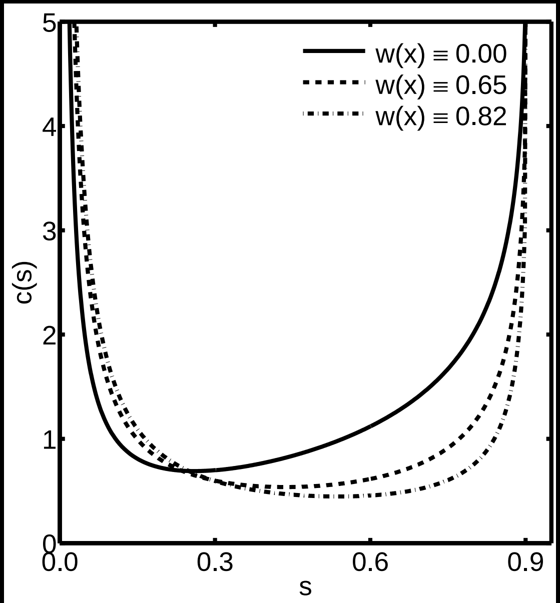




- The rate of spread is the minimum $c(s)$
- Existence of *bad* patches can halt the invasion

Invasion Model: Traveling Periodic Wave

- $0 \leq w(x) \leq 1$ and *high* deposition in *good* patches
- Dispersion relation for the speed of the wave:



- Local dispersal can slow the invasion

Invasion Model:

- Invasibility
  - *Bad* patches lower invasibility
  - Local dispersal increases invasibility

- Spread rate
  - *Bad* patches lower spread rate
  - Local dispersal lowers spread rates