

Quiz #5

M1220-1

Fall 2003

Name: _____

Score: _____

QUESTIONS: each question is worth 20 points. Give your answer and a short but complete motivation of it for full credit.

1. Decide whether the following improper integrals converge or diverge. You are NOT asked to compute the integral. However, give a short motivation for your answer!

$$\int_0^{\infty} \frac{x^{16} + 32x^8 + x^4 - 15x^3 + 12}{13x^{16} + 14x^6 + x^2 + 1} dx$$

$$\int_0^{\infty} \frac{x^{16} + 32x^8 + x^4 - 15x^3 + 12}{13x^{16} + 14x^6 + x^2 + 1} e^{-x} dx$$

2. Let $f(x)$ be a continuous one to one function defined on $(0, \infty)$, with a vertical asymptote at $x = 0$, such that

$$\int_0^1 f(x) dx = 32.$$

What can we say about

$$\int_{f(1)}^{\infty} f^{-1}(y) dy?$$

Explain your answer. (You can use pictures to do that).

3. Find the error in the following WRONG exercise:

$$\begin{aligned} & \int_{-\infty}^{+\infty} \sin(x) dx = \\ &= \lim_{b \rightarrow +\infty} \int_{-b}^{+b} \sin(x) dx = \\ &= \lim_{b \rightarrow +\infty} -\cos(x) \Big|_{-b}^{+b} = \\ &= \lim_{b \rightarrow +\infty} -\cos(b) + \cos(-b) = \\ &= \lim_{b \rightarrow +\infty} -\cos(b) + \cos(b) = \\ &= \lim_{b \rightarrow +\infty} 0 = 0. \end{aligned}$$

4. Write explicit formulas for the following sequences:

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$$1, -1, 1, -1, 1, -1, 1, -1, \dots$$

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$$0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$$

•

$$0, \frac{1}{8}, \frac{4}{27}, \frac{9}{64}, \frac{16}{125}, \frac{25}{216}, \dots$$

PROBLEM: the problem is worth 20 points. Show me enough work to let me know what you are doing!

1. Calculate the following limit:

$$\lim_{n \rightarrow +\infty} \frac{\sin(\frac{1}{n})(e^{\frac{1}{n}} - 1)}{\frac{1}{n^2}}.$$