

Math 1220-1 Exam 0

Functions and Limits

1. Let $f(x) = \sqrt{9-x}$ and $g(x) = x^2$. What is the domain of each of the following?

- (a) f
- (b) $f \circ g$
- (c) $g \circ f$

2. Find the indicated limit or state that it does not exist.

(a)

$$\lim_{u \rightarrow 2} \frac{u^2 - 4}{u + 2}$$

(b)

$$\lim_{x \rightarrow 0} \frac{\cos x}{x}$$

The Derivative

3. Find the indicated derivative by using the rules we developed in Calculus I.

(a)

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x^2 + 9}} \right)$$

(b)

$$D_\theta^2(\sin \theta + \cos^4 \theta)$$

4. Assume that f , g , and h are all differentiable. Find $f'(t)$ if $f(t) = g(h(t)) + g^2(t)$.

5. Find $\frac{dy}{dx}$ in each case.

- (a) $(x+2)^2 + y^2 = 4$
- (b) $x^5 + y^5 = x^5 y^5$
- (c) $x \tan(xy) = x^2 + 1$

Applications of the Derivative

6. An object is projected directly upward from the ground with an initial velocity of 200 feet per second. Its height s at the end of t seconds is approximately $s = 200t - 16t^2$ feet.

- (a) When does it reach its maximum height and what is this height?
- (b) When does it hit the ground and with what velocity?

7. Find where the function g defined by

$$g(t) = t^3 + \frac{1}{t}$$

is increasing and where it is decreasing. Find the local extreme values of g . Find the points of inflection. Sketch the graph.

8. A page of a book is to contain 30 square inches of print. If the margins at the top, bottom, and one side are 1 inch and the margin at the other side is 2 inches, what size page would use the least paper?

The Integral

9. Evaluate the integral

$$\int_0^{\pi/2} \cos^3 x \sin x \, dx$$

10. Solve the differential equation

$$\frac{dy}{dx} = \sin x$$

subject to the initial condition $y = 2$ at $x = 0$.

11. If a particle moving on the x -axis has acceleration

$$a = 5\sqrt{t} + 2$$

at time t and if $v_0 = -3, x_0 = -11$, find its position x at $t = 4$. Assume that x is measured in feet and t in seconds.

12. Find $G'(x)$ for each function G .

(a)

$$G(x) = \int_1^x \sin^2 z \, dz$$

(b)

$$G(x) = \int_x^{x+1} f(t) \, dt$$

(c)

$$G(x) = x^2 \int_0^x f(t) \, dt$$

Applications of the Integral

13. Let R be the region bounded by the curve $y = x - x^2$ and the x -axis.

(a) Find the area of R .

(b) Find the volume of the solid S generated by revolving the region R about the x -axis.