

## Exam #3

M1220-1

Fall 2003

Name: \_\_\_\_\_

Score: \_\_\_\_\_

Each problem is worth 15 points. Give your answer and a short but complete motivation of it for full credit.

1. Determine if the following series converges or diverges.

$$\sum_3^{\infty} \frac{n^2 + 5}{n!}$$

2. Determine if the following series converges. Does it converge absolutely?

$$\sum_{28}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

3. Give a lower and upper approximation for the value of convergence of the series below, such that the error is smaller than  $1/100$ .

*This means: tell me  $a$  and  $b$  such that  $a \leq \sum \leq b$  and  $b - a \leq 1/100$ .*

$$\sum_1^{\infty} (-1)^n \frac{1}{n^3}$$

4. Let

$$a_n = \begin{cases} \frac{1}{n} & n \text{ even} \\ \frac{1}{2^n} & n \text{ odd} \end{cases}$$

Does  $\sum_1^{\infty} (-1)^n a_n$  converge?

*EXTRA CREDIT:*(+5 points) give me a correct, nice and clear motivation for your answer.

5. Determine the radius of convergence of the following power series.

$$\sum_0^{\infty} \frac{n}{3^n} x^n$$

6. Write the Mac Laurin expansion of

$$f(x) = \frac{x^2}{1-x^3}.$$

7. Use Taylor expansions to compute the following limit:

$$\lim_{x \rightarrow 0} \frac{(\sin x - x)(\sinh x - x)}{(\cos x - 1)(e^{x^2} - 1)(\ln^2(1+x))}$$