Exam #1

M1220-1 Fall 2003

Name:_____ Score: _____

Each problem is worth 10 points. Give your answer and a short but complete motivation of it for full credit.

1. Solve the following logarithmic equations:

$$\ln(x^3) = 0$$

$$\ln e^x = 3.$$

2. Determine if the following functions are invertible:

$$y = x^3 + x^5 + 4x + 1$$

$$y = x^3 + x^5 + 4x + \cos(x).$$

3. Evaluate the following limit:

$$\lim_{h \to 0} (1 + 3h)^h.$$

4. Compute the following derivative:

$$\frac{d}{dx}(2^x ln x).$$

5. A population of bacteria grows at a rate proportional to its size. If initially you have 1000 bacteria and after 10 days you have 2000, how many bacteria will you have after 25 days?

6. Set up (you are NOT required to solve it!!) the differential equation relative to the following word problem.

   You have a tank initially containing 1000 liters of salt water, with 50 kg of salt dissolved. Water flows in at a rate of 1 liter per minute and also salt is sifted in at a rate of 0.5 kg per minute. Solution flows out at a rate of 2 liters per minute. Determine the quantity of salt in the tank at a given time t.

7. Solve the following linear differential equation:

$$y' \ln(x) + \frac{y}{x} = x,$$

   with initial condition

$$y(e) = \frac{e^2}{2}.$$

8.

$$\int \cosh(\sinh(u))\cosh(u)du = ?$$

9.

$$\int \cos^3(x)dx = ?$$

10.

$$\int_0^1 \frac{dx}{x^2+3x+2} = ?$$

11. EXTRA CREDIT (this is hard, so before you spend a huge amount of time on it, make sure that you are satisfied with the rest of the test!)

$$\int x^3 e^{x^2} dx = ?$$