Integrals of Various Trig Functions

Math 1220 (Spring 2003)

Here are three types of trig integrals that we can do:

1. Integrals of the form:

$$\int \sin^n(x)dx \text{ or } \int \cos^n(x)dx$$

2. Integrals of the form:

$$\int sin^m(x)\cos^n(x)dx$$

3. Integrals of the form:

$$\int \sin(mx)\cos(nx)dx, \text{ or } \int \sin(mx)\sin(nx)dx \text{ or } \int \cos(mx)\cos(nx)dx$$

Type 1. Two subcases:

A. If $n = 2k+1$ is odd, then rewrite the integral (e.g. the first one) as:

$$\int \sin^{2k}(x)\sin(x)dx$$

and substitute:

$$\sin^2(x) = (1-\cos^2(x))$$

to get:

$$\int (1-\cos^2(x))^k\sin(x)dx$$

Now u substitute $u = \cos(x)$ to get:

$$-\int (1-u^2)^k du$$

and then multiply out and do the integral!

B. If $n = 2k$ is even, then use the half-angle identities over and over:

$$\cos^2(x) = \frac{1+\cos(2x)}{2},\ \sin^2(x) = \frac{1-\cos(2x)}{2}$$

so we can rewrite:

$$\int \cos^{2k}(x)dx = \int \left(\frac{1+\cos(2x)}{2}\right)^k dx$$

and then expand out and do the odd powers (as in (a)) and do the even powers by using half-angle identities again!

Type 2. Two subcases:

A. If m or n is odd, then rewrite the integral as in Type 1A preserving one copy of $\sin(x)$ (or $\cos(x)$) for the u substitution.

B. If both m and n are even, use half-angle formulas to bring the powers down just as in Type 1A.

Type 3. Use the following formulas:

$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin(m+n)x + \sin(m-n)x)$$

$$\sin(mx)\sin(nx) = -\frac{1}{2}(\cos(m+n)x - \cos(m-n)x)$$

$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos(m+n)x + \cos(m-n)x)$$

I'll do lots of examples in class!