First Techniques of Integration

Math 1220 (Spring 2003)

We now have a fairly long list of functions and their derivatives:

- Log functions

$$\frac{d}{dx}\ln(x) = \frac{1}{x}, \quad \frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

- Exponential functions

$$\frac{d}{dx}e^x = e^x, \quad \frac{d}{dx}a^x = a^x\ln(a)$$

- Power functions

$$\frac{d}{dx}x^a = ax^{a-1}$$

- Trig functions

$$\frac{d}{dx}\sin(x) = \cos(x), \ \frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x), \ \frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

- Inverse trig functions

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}, \ \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}, \ \frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

- Hyperbolic functions

$$\frac{d}{dx}\sinh(x) = \cosh(x), \ \frac{d}{dx}\cosh(x) = \sinh(x)$$

$$\frac{d}{dx}\tanh(x) = \text{sech}^2(x), \ \frac{d}{dx}\text{sech}(x) = -\text{sech}(x)\tanh(x)$$

- Inverse hyperbolic functions

$$\frac{d}{dx}\sinh^{-1}(x) = \frac{1}{\sqrt{x^2+1}}, \ \frac{d}{dx}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}, \ \frac{d}{dx}\text{sech}^{-1}(x) = -\frac{1}{x\sqrt{1-x^2}}$$

We want to use this list to help us evaluate integrals. The first methods for (possibly) turning a hard-looking integral into one that we can solve are:

u-substitution: Any indefinite integral that looks like:

$$\int f(u(x))u'(x)dx$$

can be simplified:

$$\int f(u(x))u'(x)dx = \int f(u)du$$

so that if we can find $\int f(u)du$, then we can find the original integral. This also works for definite integrals (but be careful with the limits!).

Completing the Square: A lot of our new derivatives involve x^2. If an integral involves a quadratic polynomial (usually in the denominator), then "completing the square" and a u-substitution might make the integral doable.

Example:

$$\int \frac{1}{x^2+2x+2}dx = \int \frac{1}{(x+1)^2+1}dx = \int \frac{1}{u^2+1}du = \tan^{-1}(u) + c$$

$$= \tan^{-1}(x+1) + c$$

using the u substitution $u = x+1, du = dx$.

Dividing Out Expressions: If an integral is given as a fraction:

$$\int \frac{f(x)}{g(x)}dx$$

and if the degree of $f(x)$ is $\geq$ the degree of $g(x)$, then long divide first.

Example:

$$\int \frac{x^2}{x+1}dx = \int x - 1 + \frac{1}{x+1}dx =$$

$$= \frac{x^2}{2} - x + \ln(x+1) + c$$

(You need to remember how to long divide polynomials!!)