Some Persepctive on Chapter 9

Math 1220 (Spring 2003)

- l'Hôpital's rule tells us that:

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

if $\lim_{x\to c} f(x) = 0 = \lim_{x\to c} g(x)$ or $\lim_{x\to c} f(x) = \pm\infty = \lim_{x\to c} g(x)$ (but not for other limits of the numerator and denominator!)

Other limits can be converted into these using tricks like:

$$\lim_{x\to c} f(x)g(x) = \lim_{x\to c}\frac{f(x)}{1/g(x)}$$

or the natural log trick:

$$\lim_{x\to c} f(x)^{g(x)} = y$$

$$\lim_{x\to c} g(x)\cdot\ln(f(x)) = \ln(y)$$

$$e^{\ln(y)} = \lim_{x\to c} f(x)^{g(x)}$$

- Improper integrals may look like:

$$\int_a^\infty f(x)dx,\ \int_{-\infty}^b f(x)dx \text{ or } \int_{-\infty}^\infty f(x)dx$$

These are computed in the ordinary way. You find the antiderivative then plug in the endpoints, except that an infinite enpoint is interpreted as a limit.

- Improper integrals may also look like:

$$\int_a^b f(x)dx$$

when the *function* $f(x)$ blows up at a or b. Again, they are computed in the ordinary way, with the (finite) enpoints intepreted as a limit, when necessary.

- Finally, if an imporper integral looks like:

$$\int_a^b f(x)dx$$

and if $f(x)$ blow up a some point c that is **between** a and b, then you need to split up the integral into two integrals:

$$\int_a^c f(x)dx + \int_c^b f(x)dx$$

and this is crucial. It may look like the integral is finite when it isn't!