

## Some Perspective on Chapter 8

Math 1220 (Spring 2003)

Chapter 8 is about techniques of integration. Let's review:

**Math 1210 Stuff:** You already learned in 1210 how to integrate:

- Polynomials
- Basic trig functions ( $\sin(x)$  and  $\cos(x)$ )

and you learned about  $u$  substitutions, which allow you to do tricky ones. One important example of a  $u$ -substitution (from Chapter 7) is:

$$\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx = - \int \frac{du}{u} = -\ln(|u|) + C = -\ln(|\cos(x)|) + C$$

for the  $u$ -substitution  $u = \cos(x)$ .

**Chapter 7 Stuff:** In Chapter 7 you learned how to integrate:

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$$\frac{1}{x}$$

- Any power and exponential function.
- The basic hyperbolic functions ( $\sinh(x)$  and  $\cosh(x)$ )
- Some special functions:

$$\frac{1}{1+x^2}, \quad \frac{1}{\sqrt{1-x^2}}, \quad \frac{1}{\sqrt{1+x^2}}, \quad \frac{1}{1-x^2}, \quad -\frac{1}{x\sqrt{1-x^2}}$$

and of course you can put these together with  $u$ -substitutions.

**Chapter 8 Stuff:** In Chapter 8 there were some new ideas.

- clever  $u$  substitutions

$$u = \sqrt[n]{ax+b}, \quad u^n = ax+b, \quad x = \frac{u^n - b}{a}, \quad nu^{n-1}du = adx$$

for doing integrals that involve  $\sqrt[n]{ax+b}$ .

- trig substitutions:

$$x = a \sin(u), \quad dx = a \cos(u)du \quad \text{when the integral involves } \sqrt{a^2 - x^2}$$

$$x = a \tan(u), \quad dx = a \sec^2(u)du \quad \text{when the integral involves } \sqrt{a^2 + x^2}$$

$$x = a \sec(u), \quad dx = a \tan(u) \sec(u)du \quad \text{when the integral involves } \sqrt{x^2 - a^2}$$

- completing the square to convert

$$x^2 + bx + c \text{ into } \left(x + \frac{b}{2}\right)^2 + \left(c - \frac{b^2}{4}\right)$$

whenever you see it in an integral

- long division to simplify fractions of polynomials with big numerator:

$$\frac{p(x)}{q(x)} \text{ and turn them into } f(x) + \frac{r(x)}{q(x)}$$

where  $f(x)$  is the quotient polynomial and  $r(x)$  is the remainder polynomial.

- partial fractions to simplify fractions of polynomials:

$$\frac{r(x)}{q(x)}$$

(after you've long divided! if necessary) if you can factor the denominator.

- integration by parts to convert:

$$\int uv' dx = uv - \int u'v dx$$

whenever such a conversion will make the integral easier.

There is some “art” to the  $u$ -substitution and integration by parts. You need to guess the choice of  $u$  (and  $v$ ) in advance. Only practice will help you to become good at the guesswork!