

MATHEMATICS 1250-1 Honors Calculus I

What is Calculus? A Mission Statement

Arithmetic, algebra and calculus are the basic tools of mathematics. Arithmetic is about the operations with numbers: addition, subtraction, multiplication, division, exponentiation and logarithms. Algebra is about handling “unknown” quantities by setting up equations involving one or more variables and solving them. Calculus is about quantifying “rates of change” by means of the derivative and its inverse, the integral.

I do not include geometry, the study of shapes, as a tool. Geometry is the first great application of the three tools. The Pythagorean theorem, for example, is an application of algebra to the study of right triangles via the polynomial equation $a^2 + b^2 = c^2$. Calculus is employed, for example, in explaining why the same(!) constant, π , appears in the equations $c = \pi d$ and $a = \pi r^2$ describing, respectively, the circumference of a circle in terms of its diameter, and its area in terms of its radius.

One could argue that logic, which is the foundation of the mathematical proof and which figures very prominently in classical Euclidean geometry (but sadly neglected in many modern high school geometry classes) is a fourth mathematical tool, but I think logic is extra-mathematical. Logic is essential not just to mathematics, but to philosophy, law, and virtually every form of written and verbal communication. It is a crime that we do not teach logic and logical thinking in any systematic way to our children. But I digress.

Since change is an essential feature of nature, calculus is essential in any description of nature. Every science uses the language of mathematics. One of the most important functions of science, from biology to chemistry to physics to meteorology, is its ability to make predictions. This involves solving the equations that model nature, which are inevitably “differential equations,” that is, equations involving calculus. Solving difficult differential equations are among the most thorny problems in all of mathematics.

My goal is to explain to you not just how calculus works, but what it means. In other words, I want you to be able to solve calculus problems but also to understand what you are doing when you solve them. I want to show you some of the standard applications of calculus, and I want you to recognize the situations where calculus is useful. In short, I want you to emerge from my class fluent with this third basic tool of mathematics.