Math 5090, Assignment 8: Chapter 14, Exercises 7, 17, 19, 21.

7. (a) There are $n = 60$ observations. None is exactly equal to 5.20. The number that are smaller than 5.20 is 22. For the given alternative hypothesis, the p-value is $P(\text{BIN}(60, 1/2) \leq 22) = 0.02595$ (from a spreadsheet). But we were asked to use the normal approximation

$$P(\text{BIN}(60, 1/2) \leq 22) \approx \Phi\left(\frac{22.5 - 60(1/2)}{\sqrt{60(1/2)(1/2)}}\right) = \Phi(-1.9465) = 0.02640.$$

In either case this is less than 0.05, so we reject H_0.

(b) Derivation using (14.3.8): $P(X_{i:n} \leq x_{0.5} \leq X_{j:n}) = P(\text{BIN}(n, 1/2) \leq j - 1) - P(\text{BIN}(n, 1/2) \leq i - 1)$, and $P(\text{BIN}(60, 1/2) \leq 36) = 0.9538$ and $P(\text{BIN}(60, 1/2) \leq 23) = 0.4623$. Thus, $(X_{24:60}, X_{37:60}) = (5.22, 5.25)$ is the desired interval.

(c) $P(X_{k:n} \leq x_p) = P(\text{BIN}(n, p) \geq k) = 1 - P(\text{BIN}(n, p) \leq k - 1)$, so $X_{k:60}$ is a 95% lower confidence limit for $x_{0.25}$ if $P(\text{BIN}(60, 0.25) \leq k - 1) \leq 0.05$. But $k = 10$ is the largest k for which this holds. Since $X_{10:60} = 5.13$, 5.13 is a 95% LCB for $x_{0.25}$.

17. (a) Pearson's r is the statistic at the top of page 487. The approximate t distribution is stated at the bottom of page 488. We can compute $r = 0.1646$ and $\sqrt{n-2}\, r/\sqrt{1 - r^2} = 0.4720$. This does not exceed $t_{0.95}(8) = 1.860$ in absolute value, so we fail to reject H_0, assuming a two-sided alternative.

(b) We apply Pearson's r to the ranks (using average ranks in the case of a tie) to get Spearman's R_s, and $R_s = 0.2080$. (Remark: Most students got 0.215 using the formula involving $\sum d_i^2$, but that formula was derived assuming the ranks are $1, 2, \ldots, n$. In this case we had ties, so that assumption was not met.) We fail to reject H_0 at the $\alpha = 0.10$ level by Table 14 (the p-value is between 0.560 and 0.584, double the values in the table, because the test is two-sided). The large-sample approximation is $\sqrt{n-2}\, R_s/\sqrt{1 - R_s^2} = 0.6013$. Again we fail to reject H_0 since $t_{0.95}(8) = 1.860$.

19. $R_s = 0.636363$, which has p-value $0.027 < 0.10$ by Table 14. Alternatively $\sqrt{n-2}\, R_s/\sqrt{1 - R_s^2} = 2.3333 > t_{0.90}(8) = 1.397$. In either case we reject H_0.

21. See Example 14.8.3. We find that $n = 23$ and $\sum d_i^2 = 2703$, so $R_s = -0.3355$. This gives $t = \sqrt{n-2}\, R_s/\sqrt{1 - R_s^2} = -1.6319$. This is less than $-t_{0.90}(21) = -1.323$, so we reject H_0 and conclude there is a downward trend.