Math 5090, Assignment 6: Chapter 13, Exercise 20. Chapter 14, Exercises 1, 2, 5.

20. (a) $CM = 1/(12 \cdot 40) + (1 - e^{-0.15/100} - (1-0.5)/40)^2 + \cdots + (1 - e^{-409.97/100} - (40-0.5)/40)^2 = 0.06618$, using a spreadsheet. At level 0.10 we fail to reject H_0 because the test statistic does not exceed the critical value 0.347.

(b) We estimate θ using the MLE $\hat{\theta} = \overline{x} = 93.11975$. Replacing 100 by this estimate we find that $\widehat{CM} = 0.04336$. We must multiply this by $1+0.16/n$ with $n = 40$. We get 0.04353. At level 0.10 we fail to reject H_0 because 0.04353 does not exceed the critical value 0.177.

(c) Here we have to use the censored sampling formula (13.8.1). $CM = 1/(12 \cdot 40) + (1 - e^{-0.15/100} - (1-0.5)/40)^2 + \cdots + (1 - e^{-66.71/100} - (20-0.5)/40)^2 = 0.01250$. At level 0.10 we fail to reject H_0 because the test statistic does not exceed the critical value 0.189. (See Table 9.)

(d) $D^+ = \max(1/40 - (1 - e^{-0.15/100}), \ldots, 40/40 - (1 - e^{-409.97/100})) = 0.1057$ and $D^- = \max(1 - e^{-0.15/100} - 0/40, \ldots, 1 - e^{-409.97/100} - 39/40) = 0.0692$, hence $D = \max(D^+, D^-) = 0.1057$. Now by Table 11, we must multiply D by $\sqrt{n} + 0.12 + 0.11/\sqrt{n}$ with $n = 40$. We get 0.6830, which does not exceed the critical value 1.224, so we fail to reject H_0 at the 0.10 level.

1. (a) We simply count the number of observations that are less than 0.5. There are 10. The critical region is the left tail of the BIN$(20, 1/2)$ distribution. So the p-value is $P(\text{BIN}(20, 1/2) \leq 10) = 0.5881$ (Table 1 or from a spreadsheet). We fail to reject H_0 at level 0.10.

(b) Here we count the number of observations that are less than 0.25. There are only 2. The critical region is the left tail of the BIN$(20, 1/2)$ distribution. So the p-value is $P(\text{BIN}(20, 1/2) \leq 2) = 0.000201$ (Table 1 or from a spreadsheet). We reject H_0 at level 0.10. I graded this problem wrong, thinking that BIN$(20, 1/2)$ should be BIN$(20, 1/4)$. If you lost a point on this but shouldn't have, please let me correct it. If you gained a point but shouldn't have, you don't have to tell me.

2. We must count the number of incomes that are less than 24,800. We find that there are 13. The critical region is the right tail of the BIN$(20, 1/2)$ distribution. So the p-value is $P(\text{BIN}(20, 1/2) \geq 13) = 1 - P(\text{BIN}(20, 1/2) \leq 12) = 1 - 0.8684 = 0.1316$ (Table 1 or from a spreadsheet). We fail to reject H_0 at level 0.10.

5. The problem concerns the "first bus motor failure data", of which there are 191 observations. The number with fewer than 40,000 miles is 17. We must use the left tail of the BIN$(191, 1/4)$ distribution. So the p-value is $P(\text{BIN}(191, 1/4) \leq 17) = 1.2 \times 10^{-8}$ (from a spreadsheet). We reject H_0 at level 0.01. We could also use a normal approximation of the binomial to get the p-value, $\Phi((17.5 - 191(1/4))/\sqrt{191(1/4)(3/4)}) = \Phi(-5.05) = 2.21 \times 10^{-7}$, with the last step from a spreadsheet. The normal approximation is not very accurate here.