Math 5090, Assignment 10: Chapter 15, Exercises 14, 15, 16.

14.

$$\begin{aligned} E[e^{t_1\hat\beta_0+t_2\hat\beta_1}] &= E[e^{t_1(\hat\beta_c-\hat\beta_1\overline{x})+t_2\hat\beta_1}] \\ &= E[e^{t_1\hat\beta_c}]E[e^{(-t_1\overline{x}+t_2)\hat\beta_1}] \\ &= e^{t_1(\beta_0+\beta_1\overline{x})+(1/2)t_1^2\sigma^2/n}e^{(-t_1\overline{x}+t_2)\beta_1+(1/2)(-t_1\overline{x}+t_2)^2\sigma^2/S_{xx}} \\ &= e^{\beta_0t_1+\beta_1t_2+(1/2)\sigma^2[(\sum x_i^2/(nS_{xx}))t_1^2+(1/S_{xx})t_2^2+2(-\overline{x}/S_{xx})t_1t_2]}. \end{aligned}$$

15. (a) $\hat\beta_0 \pm t_{0.975}(8)\sqrt{\tilde\sigma^2\sum x_i^2/(nS_{xx})} = (1.10297, 1.26188)$.
(b) $\hat\beta_1 \pm t_{0.975}(8)\sqrt{\tilde\sigma^2/(S_{xx})} = (-1.53876, -1.09154)$.
(c) $(8\tilde\sigma^2/\chi^2_{0.975}(8), 8\tilde\sigma^2/\chi^2_{0.025}(8)) = (0.000885063, 0.00711704)$.
(d) $(\hat\beta_0-1)/\sqrt{\tilde\sigma^2\sum x_i^2/(nS_{xx})} = 5.29457$, and $t_{0.995}(8) = 3.255$, so reject H_0.
(e) $(\hat\beta_1+1.5)/\sqrt{\tilde\sigma^2/S_{xx}} = 1.90625$, and $t_{0.95}(8) = 1.860$, so reject H_0.
(f) $8\tilde\sigma^2/(0.05)^2 = 6.20606$, which lies inside the interval $(\chi^2_{0.05}(8), \chi^2_{0.95}(8)) = (2.73, 15.51)$, so fail to reject H_0.

16. (a) For $H_0:\beta_0 \le \beta_{00}$ vs. $H_a:\beta_0 > \beta_{00}$, reject H_0 if $(\hat\beta_0-\beta_{00})/\sqrt{\tilde\sigma^2\sum x_i^2/(nS_{xx})} \ge t_{1-\alpha}(n-2)$.
For $H_0:\beta_0 \ge \beta_{00}$ vs. $H_a:\beta_0 < \beta_{00}$, reject H_0 if $(\hat\beta_0-\beta_{00})/\sqrt{\tilde\sigma^2\sum x_i^2/(nS_{xx})} \le -t_{1-\alpha}(n-2)$.
(b) For $H_0:\beta_1 \le \beta_{10}$ vs. $H_a:\beta_1 > \beta_{10}$, reject H_0 if $(\hat\beta_1-\beta_{10})/\sqrt{\tilde\sigma^2/S_{xx}} \ge t_{1-\alpha}(n-2)$.
For $H_0:\beta_1 \ge \beta_{10}$ vs. $H_a:\beta_1 < \beta_{10}$, reject H_0 if $(\hat\beta_1-\beta_{10})/\sqrt{\tilde\sigma^2/S_{xx}} \le -t_{1-\alpha}(n-2)$.
(c) For $H_0:\sigma^2 \le \sigma_0^2$ vs. $H_a:\sigma^2 > \sigma_0^2$, reject H_0 if $(n-2)\tilde\sigma^2/\sigma_0^2 \ge \chi^2_{1-\alpha}(n-2)$.
For $H_0:\sigma^2 \ge \sigma_0^2$ vs. $H_a:\sigma^2 < \sigma_0^2$, reject H_0 if $(n-2)\tilde\sigma^2/\sigma_0^2 \le \chi^2_\alpha(n-2)$.