Name

Suppose 100 students from the University of Utah will participate in an NZT trial. Each student will take an IQ test without NZT and then again on a different day with NZT. For simplicity, suppose we know that the true standard deviation of differences is 10. Using a two-sided alternative to test the null hypothesis that NZT does not improve IQ, find the power of the test if NZT actually improves IQ by 1 point (on average). Note that this is all calculated prior to conducting the experiment.

$D_i \sim \text{iid}\ (\mu, 10^2)$

Reject if $\dfrac{\bar{d}}{\frac{10}{\sqrt{100}}} = \bar{d}$ is not between $z_{\alpha/2}$ and $z_{1-\frac{\alpha}{2}}$.

$$\pi(1) = P(\bar{D} < z_{\alpha/2} \mid \mu = 1) + P(\bar{D} > z_{1-\alpha/2} \mid \mu = 1)$$

$$= P(\bar{D} - 1 < z_{\alpha/2} - 1 \mid \mu = 1) + P(\bar{D} - 1 > z_{1-\alpha/2} - 1 \mid \mu = 1)$$

$$\boxed{\approx \Phi(z_{\alpha/2} - 1) + 1 - \Phi(z_{1-\alpha/2} - 1)}$$

(11.19 in the textbook)

19. Consider independent random samples from two normal distributions, $X_i \sim N(\mu_1, \sigma_1^2)$ and $Y_j \sim N(\mu_2, \sigma_2^2)$; $i = 1, \ldots, n_1, j = 1, \ldots, n_2$. Assuming that μ_1 and μ_2 are known, derive a $100(1-\alpha)\%$ confidence interval for σ_2^2/σ_1^2 based on sufficient statistics.

$$\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2(n_1-1)$$

$$\frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2(n_2-1)$$

$$\frac{(n_1-1)S_1^2}{(n_2-1)S_2^2} \cdot \frac{\sigma_2^2}{\sigma_1^2} \frac{(n_2-1)}{(n_1-1)} = \frac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2} \sim F(n_1-1, n_2-1)$$

$$1-\alpha = P\left(f_{\alpha/2}(n_1-1, n_2-1) < \frac{S_1^2 \sigma_2^2}{S_2^2 \sigma_1^2} < f_{1-\alpha/2}(n_1-1, n_2-1)\right)$$

$$= P\left(\frac{S_2^2}{S_1^2} f_{\alpha/2}(n_1-1, n_2-1) < \frac{\sigma_2^2}{\sigma_1^2} < \frac{S_2^2}{S_1^2} f_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)\right)$$

A $100(1-\alpha)\%$ CI for $\frac{\sigma_2^2}{\sigma_1^2}$ is:

$$\boxed{\left(\frac{S_2^2}{S_1^2} f_{\alpha/2}(n_1-1, n_2-1),\ \frac{S_2^2}{S_1^2} f_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)\right)}$$

(12.19a in the textbook)

19. Let $X_1, \ldots, X_n$ be a random sample from a normal distribution, $X_i \sim N(\mu, 1)$.
(a) Find a UMP test of $H_0 : \mu = \mu_0$ against $H_a : \mu < \mu_0$.

Reject H_0 if $\dfrac{\bar{x} - \mu_0}{\frac{1}{\sqrt{n}}} < z_\alpha$.

Suppose you have an EXP(θ, η) population. Derive a GLR test of the null hypothesis that θ = 4 against the alternative that θ > 4. Write down the test statistic and all associated maximum likelihood estimates. Then state the condition under which you will reject the null hypothesis in order to have a test of approximate size 0.05.

$\hat{\theta} = \max(\bar{x} - x_{1:n}, 4)$

$\hat{\eta} = x_{1:n}$

$\hat{\theta}_0 = 4$

$\hat{\eta}_0 = x_{1:n}$

$$\lambda = \frac{\prod_{i=1}^{n} \frac{1}{4} e^{-\frac{(x_i - x_{1:n})}{4}}}{\prod_{i=1}^{n} \left(\frac{1}{\max(\bar{x} - x_{1:n}, 4)}\right) e^{-\frac{(x_i - x_{1:n})}{\max(\bar{x} - x_{1:n}, 4)}}}$$

Reject H_0 if $-2\ln(\lambda) > \chi^2_{0.95}(1)$.

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The following table shows the number of m&ms of each type and color that were eaten by 3rd graders in an experiment.

	Red	Blue	Green	Yellow
Peanut	13 ($\cancel{x}$)	17	11	16
Peanut butter	14	19	14	15
Almond	15	16	17	19
Plain	13	17	17	16

Test whether there is a relationship between type and color of m&m. In particular, what is the expected count of red peanut m&ms, and what is the p-value? You don't need to write out the entire test statistic.

Expected count of red peanut m&ms is

$\left(\frac{13+14+15+13}{n}\right)\left(\frac{13+17+11+16}{n}\right) n$, where n is the sum of all counts in the table.

Expected count $= \frac{55}{249} \cdot \frac{57}{\cancel{249}} \cdot \cancel{249} = 12.59.$

p-value $= P\left(\chi^2(9) \geq t\right)$ where

$t = \sum \frac{(o_i - e_i)^2}{e_i}.$

(6.

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	Red	Blue	Green	Yellow
Peanut	13	17	11	16
~~Peanut butter~~	~~14~~	~~19~~	~~14~~	~~15~~
~~Almond~~	~~15~~	~~16~~	~~17~~	~~19~~
Plain	13	17	17	16

Test whether there the proportion of blue m&ms is the same for Plain and Peanut m&ms. In particular, what is the expected count of plain blue m&ms, and what is the p-value? You don't need to write out the entire test statistic.

	Blue	Non-Blue
Peanut	17	13+11+16=40
Plain	17	13+17+16=46

$$\hat{p} = \frac{17+17}{\text{total}} = \frac{17+17}{17+17+40+46} = \frac{34}{130} = .2615$$

Expected plain blues $= (17+46)\frac{34}{130} = 16.48$

p-value $= P(\chi^2(1) \geq t)$

where t is the outcome of the test statistic.

Circle only the statements that are true. You will get 1 point for 6 correct answers; 2 points for 7 correct; 3 points for 8 correct; 4 points for 9 correct. In other words, you will get max(# correct – 5, 0) points for this problem.

1. The outcome of a random interval that contains the parameter of interest with 95% probability is called a confidence interval for the parameter of interest.
2. If two researchers use the exact same data from an experiment and report different confidence intervals for the same parameter, then they may both be right.
3. A one-sided upper confidence interval is most appropriate when a drug company wants to establish (with 95% confidence) that no more than 5% of people will have an allergic reaction to a birth control patch that they are developing.
4. MLE – θ is a pivotal quantity for θ if MLE is a maximum likelihood estimator for θ and θ is a location parameter.
5. Pivotal quantities are functions of the random sample and the parameter of interest.
6. If (1, 2) is a 95% confidence interval for μ, then (1, 4) is a 95% confidence interval for μ^2.
7. If the null hypothesis is not rejected, then you should conclude that the alternative hypothesis is plausible.
8. The Neyman-Pearson lemma provide a method to obtain the most powerful test of a simple null hypothesis against a two-sided alternative.
9. The generalized likelihood ratio test provides a reasonable test, whose test statistic is the likelihood ratio, and whose distribution under the null hypothesis is $\chi^2(r)$, where r parameters are fixed under the null.