Quiz 8, Attempt 2

29. Let $X_1, \ldots, X_n$ be a random sample from a distribution with pdf $f(x; \theta) = 1/\theta$ if $0 \le x \le \theta$ and zero otherwise. Derive the GLR test of $H_0 : \theta = \theta_0$ versus $H_a : \theta \neq \theta_0$.

$$\frac{\prod \left(\frac{1}{\theta_0}\right) 1\{0 \le x \le \theta_0\}}{\left(\frac{1}{\hat{\theta}}\right)^n 1\{x_{n:n} \le \hat{\theta}\}} = \left(\frac{\hat{\theta}}{\theta_0}\right)^n 1\{x_{n:n} \le \theta_0\}$$

$$= \left(\frac{x_{n:n}}{\theta_0}\right)^n.$$

$$L(\theta) = \prod \frac{1}{\theta} 1\{x_i \le \theta\} = \frac{1}{\theta^n} 1\{x_{n:n} \le \theta\}$$

$\hat{\theta} = x_{n:n}$ is the MLE for θ

Reject if $-2n \log\left(\frac{x_{n:n}}{\theta_0}\right) > \chi^2_{.95}(1)$

for a test of size 5%.