Name:

Quiz 8, Attempt 1

Suppose a population is exponentially distributed. Find the p-value of the generalized likelihood ratio test of H$_0$: $\mu = 5$ against H$_a$: $\mu > 5$. Express your answer in terms of a probability statement involving a known distribution and the outcome of the generalized likelihood ratio. As discussed in class, please write down the generalized likelihood ratio and compute any estimates needed.

$$\lambda = \frac{f(\underline{x};5)}{f(\underline{x};\hat{\mu})} = \frac{\prod \frac{1}{5} e^{-\frac{x_i}{5}}}{\prod \frac{1}{\hat{\mu}} e^{-x_i/\hat{\mu}}} = \left(\frac{\hat{\mu}}{5}\right)^n e^{n\bar{x}\left(\frac{1}{\hat{\mu}}-\frac{1}{5}\right)}$$

where $\hat{\mu} = \max(5, \bar{x})$.

$$= \begin{cases} 1 & \text{if } \bar{x} \le 5 \\ \left(\frac{\bar{x}}{5}\right)^n e^{n - \frac{n\bar{x}}{5}} & \text{if } \bar{x} > 5 \end{cases}$$

Reject H$_0$ if $-2\log(\lambda) > \chi^2_{1-\alpha}(1)$ for a test with type 1 error rate of approx α.

P-value $= P\left(\chi^2(1) \ge -2\log(\lambda)\right)$.

Quiz 6, Attempt 2

For a random sample of size N = 11 from a N(μ, σ² = 7) distribution, derive a testing procedure to determine whether it is plausible that the population mean is 2. Use a two-sided alternative and a type 1 error rate of 13%.

Part 1: Complete the sentence. I will reject the null hypothesis if

$$\left|\frac{\bar{x}-2}{\sqrt{\frac{7}{11}}}\right| > z_{0.935}$$

Part 2: Express the power of the test as a function of μ.

$$\pi(\mu) = P\left(\frac{\bar{X}-2}{\sqrt{\frac{7}{11}}} > z_{.935} \,\middle|\, \mu\right) + P\left(\frac{\bar{X}-2}{\sqrt{\frac{7}{11}}} < z_{.065} \,\middle|\, \mu\right)$$

$$= P\left(\frac{\bar{X}-\mu}{\sqrt{\frac{7}{11}}} > z_{.935} + \frac{2-\mu}{\sqrt{\frac{7}{11}}} \,\middle|\, \mu\right) + P\left(\frac{\bar{X}-2}{\sqrt{\frac{7}{11}}} < z_{.065} + \frac{2-\mu}{\sqrt{\frac{7}{11}}} \,\middle|\, \mu\right)$$

$$= 1 - \Phi\left(z_{.935} + \frac{2-\mu}{\sqrt{\frac{7}{11}}}\right) + \Phi\left(z_{.065} + \frac{2-\mu}{\sqrt{\frac{7}{11}}}\right).$$

What is the power of the test if μ = 2?

0.13