Assume $EXP(\theta, \eta)$ population.

Derive a GLR test of $H_0: \theta = 4$ vs. $H_a: \theta > 4$.

Step 1: Find MLEs under $H_0 \cup H_a$: $\theta \geq 4$, $\eta \in \mathbb{R}$.

$$L(\theta, \eta) = \prod \frac{1}{\theta} e^{-\frac{(x_i - \eta)}{\theta}} \mathbb{1}\{x_i \geq \eta\}$$

$$= \frac{1}{\theta^n} \exp\left[\frac{-1}{\theta}\left(\sum x_i - n\eta\right)\right] \mathbb{1}\{x_{1:n} \geq \eta\}$$

No matter what $\theta > 0$ is, $\hat{\eta} = x_{1:n}$ is the the best choice for η.

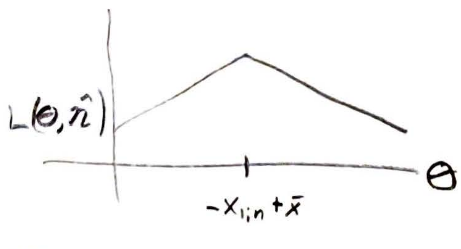
Now, $$L(\theta, \hat{\eta}) = \frac{1}{\theta^n} e^{\frac{-1}{\theta}[n\bar{x} - n\hat{\eta}]}$$

$$\ell(\theta, \hat{\eta}) = -n\ln(\theta) - \frac{1}{\theta}[n\bar{x} - n\hat{\eta}]$$

$$\frac{d}{d\theta} \ell(\theta, \hat{\eta}) = \frac{-n}{\theta} + \frac{n\bar{x} - n\hat{\eta}}{\theta^2} := 0$$

$$\Rightarrow -n\theta + n\bar{x} - n\hat{\eta} = 0$$

$$\Rightarrow \theta = -\hat{\eta} + \bar{x} = -x_{1:n} + \bar{x}$$ is a critical point.



$$\boxed{\begin{aligned} \hat{\theta} &= \max(4, -x_{1:n} + \bar{x}) \\ \hat{\eta} &= x_{1:n} \end{aligned}}$$

Step 2: Find MLEs under H_0.

$$\boxed{\begin{array}{l}\hat{\theta}_0 = 4 \\ \hat{\eta}_0 = X_{1:n}\end{array}}$$

Step 3: Define λ.

$$\lambda(\underline{x}) = \frac{\prod_{i=1}^{n} \frac{1}{\hat{\theta}_0} e^{-\frac{(x_i - \hat{\eta}_0)}{\hat{\theta}_0}}}{\prod_{i=1}^{n} \frac{1}{\hat{\theta}} e^{\frac{-x_i - \hat{\eta}}{\hat{\theta}}}}.$$

Step 4: The test.

Reject H_0 if $-2\ln(\lambda) > \chi^2_{.95}(1)$ for a test with $\alpha = 0.05$.