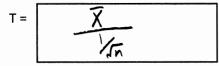
Throughout this exam, put all of your answers in the boxes provided. Each page represents a standalone problem, but there may be multiple parts within the page.

Suppose you have a normally distributed population with a variance of 1. What test statistic would make a good test for the hypotheses below? (1 point)



Construct a rejection region for a test of size 16% for a null hypothesis of a mean less than or equal to zero against a one sided alternative that the mean is greater than zero. (1 point)

Reject the null hypothesis if the outcome, t, of the test statistic, T, is in this interval:

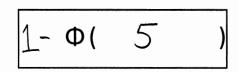
(<del>Z</del>.84 , Ø)

What is the probability of type II error if the population mean is 7? (1 point)

$$P(T \leq Z_{.84} \mid M = 7)$$
  
=  $P(\frac{X-7}{\sqrt{n}} \leq Z_{.84} - 7\sqrt{n} \mid M = 7) = \bigoplus (Z_{.84} - 7\sqrt{n})$ 

Compute the p-value if the outcome of the test statistics is 5. (1 point)

$$p$$
-value =  $P(T \ge 5 | M = 0)$ 



(1 point) Suppose you have an exponentially distributed population with a mean of either 1 or 2 and you have a sample of size 10 from that population. A uniformly most powerful test of size 10% of the null hypothesis that the mean is 1 against the alternative that the mean is 2 is to reject the null hypothesis if

$$X > \underbrace{\frac{\chi_{.90}^{2}(20)}{20}}_{ZO}$$

$$\lambda = \underbrace{\frac{10}{11}}_{Ii} \frac{1}{\lambda_{0}} e^{-\chi_{i}/\lambda_{0}}_{Ii} = (\frac{\lambda_{1}}{\lambda_{0}})^{10} e^{(-\frac{1}{\lambda_{0}} + \frac{1}{\lambda_{1}})\sum_{i=1}^{10} \chi_{i}}_{Ii}$$

$$Reject if \quad Z_{Xi} \quad is \quad large.$$

$$\frac{Z \sum \chi_{i}}{1} \quad H_{0} \chi^{2}(20)_{I}$$

$$Reject if \quad Z_{Xi} \quad \chi^{2}(20)$$

(1 point) A uniformly most powerful test of size 10% would be to reject the null hypothesis if

$$\mathbf{x} > \frac{\mathcal{X}^{2}_{.10}(20)}{20}.$$

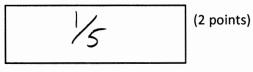
Suppose you have a population with a Poisson distribution. A generalized likelihood ratio test of size 5% of the null hypothesis that the mean is 12 against the 2-sided alternative is to reject the null hypothesis if

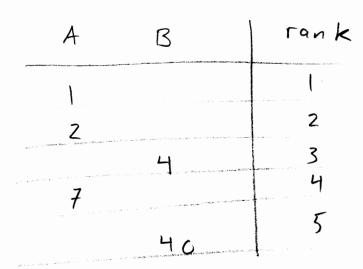
-2log(
$$\lambda$$
) >  $\chi^{2}(1)$  (1 points)  
Where  $\lambda = \left[ e^{n(\bar{x}-12)} \left(\frac{12}{\bar{x}}\right)^{n\bar{x}} \right]$  (1 points)

$$\lambda = \frac{f(\underline{x}; 1z)}{F(\underline{x}; \overline{x})} = \prod \frac{e^{-1z}}{|z|} \frac{x_i}{|x_i|} = e^{n(\overline{x}-1z)} \left(\frac{1z}{\overline{x}}\right)^{\sum x_i}$$

$$\prod \frac{e^{-\overline{x}}}{|x_i|} = \frac{x_i}{|x_i|}$$

A random sample of size 3 is obtained from Population A with outcomes of 1,2,7. A random sample of size 2 is obtained from Population B with outcomes of 4 and 40. The two samples are independent. Population A and Population B are thought to have similarly shaped distributions but with a possible shift in the mean. The p-value for a Wilcoxon rank-sum test of the null hypothesis that the means are the same against the one-sided alternative that the mean for Population A is smaller is





In a random sample of size 50 from the population of Utah voters, 30 prefer Hilary over Trump; 10 prefer Trump over Hilary, and 10 hate them both equally. Use a sign test to test the null hypothesis that Utahans are evenly split between favoring Hilary vs. Trump against the alternative that Hilary is preferred over Trump. The p-value is

(2 points)  $P(BIN(40, \frac{1}{2}) \geq 30)$ 

11. A sample of 400 people was asked their degree of support of a balanced budget and their degree of support of public education, with the following results:

Public Education	Supported Balanced Budget			
	Strong	Undecided	Weak	an an an an stàir.
Strong	100	80 .	20	200
Undecided	60	50	15	125
Weak	20	50	5	75
	180	180	40	400

We would like to test the null hypothesis that support for a balanced budget is independent of support for public education.

Under the null hypothesis, what the expected number with strong support for both is

$$400\left(\frac{180}{400}\right)\left(\frac{200}{400}\right) = 90$$

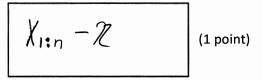
(1 point)

Using the standard chi-squared test statistic, t, for this test, the p-value is

$$P(\chi^{2}(\mathcal{H}) \geq t) \qquad (1$$

point)

Suppose you have a population whose distribution is the two-parameter exponential with location parameter  $\eta$  and scale parameter  $\theta = 1$ . A pivotal quantity for  $\eta$  is



A one-sided upper 90% confidence bound on  $\boldsymbol{\eta}$  is

$$X_{i:n} + \frac{\log(.9)}{n}$$
 (1 point)

• 
$$90 = P(a < \chi_{iin} - \mathcal{R}) = P(\mathcal{R} < \chi_{iin} - a)$$

$$=P(X_{1:n} > a + \pi) = \left[P(X_1 > a + \pi)\right]^{n} = e^{-an}$$

since 
$$P(X_1 - \pi, 7\alpha) = \int_{\alpha}^{\infty} e^{-x} dx = -e^{-x} \Big|_{\alpha}^{\infty}$$
  
=  $+e^{-\alpha}$ 

If 
$$.9 = e^{-an}$$
, then  $log(.9) = (-an)$   
=>  $a = -log(.9)$