Throughout this exam, put all of your answers in the boxes provided. Each page represents a stand-alone problem, but there may be multiple parts within the page.

Suppose you have a normally distributed population with a variance of 1. What test statistic would make a good test for the hypotheses below? (1 point)

T = $\boxed{\dfrac{\bar{X}}{1/\sqrt{n}}}$

Construct a rejection region for a test of size 16% for a null hypothesis of a mean less than or equal to zero against a one sided alternative that the mean is greater than zero. (1 point)

Reject the null hypothesis if the outcome, t, of the test statistic, T, is in this interval: $\boxed{(z_{.84}, \infty)}$

What is the probability of type II error if the population mean is 7? (1 point)

$$P(T \le z_{.84} \mid \mu = 7)$$

$$= P\left(\frac{\bar{X} - 7}{1/\sqrt{n}} \le z_{.84} - 7\sqrt{n} \;\middle|\; \mu = 7 \right) = \boxed{\Phi(z_{.84} - 7\sqrt{n})}$$

Compute the p-value if the outcome of the test statistics is 5. (1 point)

$$\text{p-value} = P(T \ge 5 \mid \mu = 0)$$

$$\boxed{1 - \Phi(5)}$$

(1 point) Suppose you have an exponentially distributed population with a mean of either 1 or 2 and you have a sample of size 10 from that population. A uniformly most powerful test of size 10% of the null hypothesis that the mean is 1 against the alternative that the mean is 2 is to reject the null hypothesis if

$\bar{x}$ > $\boxed{\dfrac{\chi^2_{.90}(20)}{20}}$.

$$\lambda = \frac{\prod_{i=1}^{10} \frac{1}{\lambda_0} e^{-x_i/\lambda_0}}{\prod \frac{1}{\lambda_1} e^{-x_i/\lambda_1}} = \left(\frac{\lambda_1}{\lambda_0}\right)^{10} e^{\left(-\frac{1}{\lambda_0} + \frac{1}{\lambda_1}\right)\sum_{i=1}^{10} x_i}$$

Reject if Σx_i is large.

$$\frac{2\Sigma X_i}{1} \overset{H_0}{\sim} \chi^2(20)$$

Reject if $2\Sigma x_i > \chi^2_{.90}(20)$

(1 point) A uniformly most powerful test of size 10% would be to reject the null hypothesis if

$\bar{x}$ > $\boxed{\dfrac{\chi^2_{.90}(20)}{20}}$.

Suppose you have a population with a Poisson distribution. A generalized likelihood ratio test of size 5% of the null hypothesis that the mean is 12 against the 2-sided alternative is to reject the null hypothesis if

-2log(λ) >	$\chi^2_{.95}(1)$	(1 points)
Where λ =	$e^{n(\bar{x}-12)}\left(\frac{12}{\bar{x}}\right)^{n\bar{x}}$	(1 points)

$$\lambda = \frac{f(\underline{x};12)}{f(\underline{x};\bar{x})} = \frac{\prod \frac{e^{-12}\,12^{x_i}}{x_i!}}{\prod \frac{e^{-\bar{x}}\,\bar{x}^{x_i}}{x_i!}} = e^{n(\bar{x}-12)}\left(\frac{12}{\bar{x}}\right)^{\Sigma x_i}$$

A random sample of size 3 is obtained from Population A with outcomes of 1,2,7. A random sample of size 2 is obtained from Population B with outcomes of 4 and 40. The two samples are independent. Population A and Population B are thought to have similarly shaped distributions but with a possible shift in the mean. The p-value for a Wilcoxon rank-sum test of the null hypothesis that the means are the same against the one-sided alternative that the mean for Population A is smaller is

$\boxed{1/5}$ (2 points)

A	B	rank
1		1
2		2
	4	3
7		4
	40	5

$w_a = 7$

1	2	3	4	5	W_a
o	o				12
o		o			11
o			o		10
o				o	9
	o	o			10
	o		o		9
	o			o	8
		o	o		8
		o		o	7
			o	o	6

In a random sample of size 50 from the population of Utah voters, 30 prefer Hilary over Trump; 10 prefer Trump over Hilary, and 10 hate them both equally. Use a sign test to test the null hypothesis that Utahans are evenly split between favoring Hilary vs. Trump against the alternative that Hilary is preferred over Trump. The p-value is

$$\boxed{P\left(BIN\left(40, \tfrac{1}{2}\right) \geq 30\right)}$$

(2 points)

11. A sample of 400 people was asked their degree of support of a balanced budget and their degree of support of public education, with the following results:

	Supported Balanced Budget			
Public Education	Strong	Undecided	Weak	
Strong	100	80	20	200
Undecided	60	50	15	125
Weak	20	50	5	75
	180	180	40	400

We would like to test the null hypothesis that support for a balanced budget is independent of support for public education.

Under the null hypothesis, what the expected number with strong support for both is

$$400\left(\frac{180}{400}\right)\left(\frac{200}{400}\right) = 90$$

$\boxed{90}$ (1 point)

Using the standard chi-squared test statistic, t, for this test, the p-value is

$\boxed{P\,(\chi^2(\;4\;) \ge t)}$ (1 point)

Suppose you have a population whose distribution is the two-parameter exponential with location parameter η and scale parameter θ = 1. A pivotal quantity for η is

$$\boxed{X_{1:n} - \eta}$$

(1 point)

A one-sided upper 90% confidence bound on η is

$$\boxed{X_{1:n} + \frac{\log(.9)}{n}}$$

(1 point)

$$.90 = P(a < X_{1:n} - \eta) = P(\eta < X_{1:n} - a)$$

$$= P(X_{1:n} > a + \eta) = [P(X_1 > a + \eta)]^n = e^{-an}$$

since $P(X_1 - \eta > a) = \int_a^\infty e^{-x}\,dx = -e^{-x}\Big|_a^\infty$

$$= +e^{-a}$$

If $.9 = e^{-an}$, then $\log(.9) = (-an)$

$$\Rightarrow a = \frac{-\log(.9)}{n}$$