

1. Let X_1, X_2, \dots, X_n be independent identically distributed random variables with density function

$$h(t; \theta) = \begin{cases} 0, & \text{if } -\infty < t < 0 \\ \frac{1}{\theta} e^{-t/\theta}, & \text{if } 0 \leq t < \infty. \end{cases}$$

We wish to test $H_0 : \theta = \theta_0$ against $H_A : \theta \neq \theta_0$. Find a test using the generalized likelihood ratio.

2. Let X_1, X_2, \dots, X_n be independent identically distributed random variables with probability mass function

$$P\{X_i = t\} = \frac{\theta^t}{t!} e^{-\theta} I\{t = 0, 1, 2, \dots\}.$$

We wish to test $H_0 : \theta = \theta_0$ against $H_A : \theta \neq \theta_0$. Find a test using the generalized likelihood ratio.

Consider a random sample of size 37 for $N(\mu_1, 1)$ and another, independent random sample of size 51 from $N(\mu_2, \sigma^2)$. Construct a test of the null hypothesis that $\mu_1 = \mu_2$ against the two-sided alternative using the generalized likelihood ratio. Write down an expression for the p-value in terms of an appropriate distribution.

Consider a random sample of size 37 for $N(\mu_1, \sigma^2)$ and another, independent random sample of size 51 from $N(\mu_2, \sigma^2)$. Construct a test of the null hypothesis that $\mu_1 = \mu_2$ against the two-sided alternative using the generalized likelihood ratio. Write down an expression for the p-value in terms of an appropriate distribution.