

1. Let  $X$  be a random variable that is uniformly distributed,  $X \sim \text{UNIF}(0,1)$ . Find the density function of  $X(1-X)$ .

$$\begin{aligned}
 F_Y(y) &= P(Y \leq y) = P(X - X^2 \leq y) = P(-X^2 + X - y \leq 0) \\
 &= \left( P\left(X \leq \frac{1}{2} - \frac{\sqrt{1-4y}}{2}\right) + P\left(X \geq \frac{1}{2} + \frac{\sqrt{1-4y}}{2}\right) \right) \\
 &\quad \left\{ \begin{array}{l} \vdots \\ \vdots \end{array} \right. \\
 &= \left\{ \begin{array}{l} \frac{1}{2} - \frac{\sqrt{1-4y}}{2} + 1 - \frac{1}{2} - \frac{\sqrt{1-4y}}{2} \\ \vdots \end{array} \right. \\
 &= \left\{ \begin{array}{ll} 1 - \sqrt{1-4y} & \text{if } 0 \leq y \leq \frac{1}{4}, \\ 0 & \text{if } y < 0, \\ 1 & \text{if } y > \frac{1}{4}. \end{array} \right.
 \end{aligned}$$

$$f_Y(y) = -\frac{1}{2}(1-4y)^{-\frac{1}{2}}(-4) \mathbb{1}_{\{y \in (0, \frac{1}{4})\}}$$

$$= \left\{ \begin{array}{ll} \frac{2}{\sqrt{1-4y}} & y \in (0, \frac{1}{4}), \\ 0 & \text{o/w.} \end{array} \right.$$