1. Let X be a random variable with density function

$$f(x) = \begin{cases} 1/4 & \text{if } x \in (-2, 2) \\ 0 & \text{if } x \notin (-2, 2). \end{cases}$$

Compute the density function of $Y = X^2$.

$$F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(X^2 \leq y) = \mathbb{P}(-\sqrt{y} \leq X \leq \sqrt{y})$$

$$= F_X(\sqrt{y}) - F_X(-\sqrt{y}).$$

$$f_Y(y) = f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} + f_X(-\sqrt{y}) \frac{1}{2\sqrt{y}}$$

$$= \begin{cases} \frac{1}{4\sqrt{y}} & y \in (0, 4), \\ 0 & \text{o/w}. \end{cases}$$