1. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \notin [0,1] \\ \theta t^{\theta-1}, & \text{if } t \in [0,1]. \end{cases}$$

Find a moment estimator for θ.

$$\bar{X} = \int_0^1 t\,\theta t^{\theta-1}\,dt$$

$$= \int_0^1 \theta t^{\theta}\,dt$$

$$= \left.\frac{\theta t^{\theta+1}}{\theta+1}\right|_{t=0}^{t=1}$$

$$= \frac{\theta}{\theta+1}$$

$$\Rightarrow \theta = \bar{X}(\theta+1) = \bar{X}\theta + \bar{X}$$

$$\Rightarrow \theta - \bar{X}\theta = \bar{X}$$

$$\Rightarrow \theta(1-\bar{X}) = \bar{X}$$

$$\Rightarrow \theta = \frac{\bar{X}}{1-\bar{X}}$$

Note that $\mathbb{P}(\bar{X} = 1) = 0$.

Therefore, $\hat{\theta} = \dfrac{\bar{X}}{1-\bar{X}}$ is a moment estimator for θ