If $x \in (0,\Theta)$, $F_X(x) = \int_0^x \frac{2t}{\Theta^2}\,dt = \frac{t^2}{\Theta^2}\Big|_0^x = \frac{x^2}{\Theta^2}$

1. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \notin [0,\theta] \\ \frac{2t}{\theta^2}, & \text{if } t \in [0,\theta]. \end{cases}$$

Find the maximum likelihood estimator for θ and compute the bias of the maximum likelihood estimator for θ.

$$L(\Theta) = \prod \left(\frac{2x_i}{\Theta^2}\right) 1\{\Theta \geq X_{min}\}$$

$$\hat{\Theta} = X_{min}$$

$$Bias(\hat{\Theta}) = E(X_{min} - \Theta)$$

If $y \in (0,\Theta)$,

$$F_{X_{min}}(y) = \mathbb{P}(X_{min} \leq y) = (P(X_1 \leq y))^n = \left(\frac{y^2}{\Theta^2}\right)^n$$

$$f_{X_{min}}(y) = \frac{2n\, y^{2n-1}}{\Theta^{2n}} 1\{y \in (0,\Theta)\}$$

$$E(X_{min}) = \int_0^\Theta y \frac{2n}{\Theta^{2n}} y^{2n-1}\,dy = \int_0^\Theta \frac{2n}{\Theta^{2n}} y^{2n}\,dy$$

$$= \frac{y^{2n+1}\, 2n}{\Theta^{2n}(2n+1)}\Big|_0^\Theta = \frac{\cancel{\Theta^{2n}}\,\Theta\, 2n}{\cancel{\Theta^{2n}}(2n+1)}$$

$$Bias = \Theta\left[\frac{2n}{2n+1} - 1\right] = \frac{-\Theta}{2n+1}$$