

1. Let X_1, X_2, \dots, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x^2 & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of $\frac{X_{n:n}}{n^{1/2}}$.

$$P\left(\frac{X_{n:n}}{n^{1/2}} \leq y\right) = P\left(X_{n:n} \leq y\sqrt{n}\right) = \left(F(y\sqrt{n})\right)^n$$

$$= \begin{cases} \left(1 - \frac{1}{y^2 n}\right)^n & y\sqrt{n} \geq 1, \\ 0 & \text{o/w.} \end{cases}$$

$$\rightarrow \begin{cases} e^{-\frac{1}{y^2}} & y > 0, \\ 0 & \text{o/w.} \end{cases}$$