

1. Let  $X$  be a random variable with density function

$$f(x) = \begin{cases} 1/4 & \text{if } x \in (-2, 2) \\ 0 & \text{if } x \notin (-2, 2). \end{cases}$$

Compute the density function of  $Y = X^3$ . Use the cdf technique and the transformation method and compare the results (if both techniques are appropriate). Otherwise, state why one or both of the techniques are not appropriate.

$$F_Y(y) = \mathbb{P}(X^3 \leq y) = \mathbb{P}(X \leq y^{1/3}) = F_X(y^{1/3})$$

$$f_Y(y) = f_X(y^{1/3}) \frac{1}{3} y^{-2/3} = \begin{cases} \frac{1}{12y^{2/3}} & y \in (-8, 8), \\ 0 & \text{o/w.} \end{cases}$$

$$f_Y(y) = f_X(x(y)) \left| \frac{d}{dy} x(y) \right| \mathbb{1}_{\{y \in (-8, 8)\}}$$

$$= \frac{1}{4} \frac{1}{3} y^{-2/3} \mathbb{1}_{\{y \in (-8, 8)\}}$$

$$= \begin{cases} \frac{1}{12y^{2/3}} & y \in (-8, 8), \\ 0 & \text{o/w.} \end{cases}$$