

1. Let X be a random variable with density function

$$f(x) = \begin{cases} 1/2 & \text{if } x \in (1, 3) \\ 0 & \text{if } x \notin (1, 3). \end{cases}$$

Compute the density function of $Y = (X - 1)^2$.

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P((X-1)^2 \leq y) = P(-\sqrt{y} \leq X-1 \leq \sqrt{y}) \\ &= P(1-\sqrt{y} \leq X \leq 1+\sqrt{y}) \\ &= F_X(1+\sqrt{y}) - F_X(1-\sqrt{y}) \end{aligned}$$

$$f_Y(y) = f_X(1+\sqrt{y}) \frac{1}{2\sqrt{y}} + f_X(1-\sqrt{y}) \frac{1}{2\sqrt{y}}$$

$$= \begin{cases} \frac{1}{4\sqrt{y}} & y \in (0, 4), \\ 0 & \text{o/w.} \end{cases}$$