

1. Let  $X$  be a random variable with density function

$$f(x) = \begin{cases} 2x & \text{if } x \in (0, 1) \\ 0 & \text{if } x \notin (0, 1). \end{cases}$$

Compute the moment generating function of  $X$  and the moment generating function of  $2X$ .

$$M_X(t) = E(e^{tX}) = \int_0^1 e^{tx} 2x \, dx$$

$$\begin{array}{ccc} 2x & & e^{tx} \\ & \swarrow + & \\ 2 & & \frac{e^{tx}}{t} \\ & \searrow - & \\ 0 & & \frac{e^{tx}}{t^2} \end{array}$$

$$= \left\{ \left( \frac{2x e^{tx}}{t} - \frac{2e^{tx}}{t^2} \right) \right\}_{x=0}^{x=1} = \left\{ \left( \frac{2e^t}{t} - \frac{2e^t}{t^2} \right) + \frac{2}{t^2} \right\} \begin{matrix} t \neq 0 \\ t = 0 \end{matrix}$$

$$M_{2X}(t) = E(e^{t2X}) = M_X(2t) = \left\{ \frac{2e^{2t}}{2t} - \frac{2e^{2t}}{(2t)^2} + \frac{2}{(2t)^2} \right\} \begin{matrix} t \neq 0 \\ t = 0 \end{matrix}$$