

1. Let X_1, X_2, \dots, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Find the cumulative distribution function of $X_{n:n}$.

$$\mathbb{P}(X_{n:n} \leq y) = \left(F_X(y)\right)^n = \begin{cases} (1 - e^{-y})^n & y \geq 0, \\ 0 & y < 0. \end{cases}$$