

1. Let X_1 and X_2 be independent identically distributed random variables with density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Compute the density function of (Y_1, Y_2) , where $Y_1 = X_1$ and $Y_2 = X_1 + 2X_2$.

$$x_1 = y_1$$

$$x_2 = \frac{y_2 - y_1}{2}$$

$$J = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$|\det(J)| = \frac{1}{2}$$

$$f_{Y_1, Y_2}(y_1, y_2) = \frac{1}{2} f_{X_1, X_2}\left(y_1, \frac{y_2 - y_1}{2}\right) = \begin{cases} \frac{1}{2} e^{-y_1} e^{-\frac{(y_2 - y_1)}{2}} & y_1 > 0 \\ & y_2 > y_1 \\ 0 & \text{o/w.} \end{cases}$$

$$= \begin{cases} \frac{1}{2} e^{-y_1/2 - y_2/2} & y_2 > y_1 > 0, \\ 0 & \text{o/w.} \end{cases}$$