

1. Let X_1 and X_2 be independent random variables with density functions

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

and

$$g(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0, \end{cases}$$

respectively. Compute the density function of $X_1 + X_2$.

$$\begin{aligned} f_{X_1+X_2}(y) &= \int_{\mathbb{R}} f(y-x)g(x)dx = \int_{\mathbb{R}} e^{-(y-x)} \mathbb{1}\{y-x \geq 0\} e^{-x} \mathbb{1}\{x \geq 0\} dx \\ &= \mathbb{1}\{y > 0\} \int_0^y e^{-y} dx = \begin{cases} ye^{-y} & y > 0, \\ 0 & \text{o/w.} \end{cases} \end{aligned}$$