1. Let $N_1, \ldots, N_5$ be independent normal random vaiables with zero mean. Asuume $EN_1^2 = EN_2^2 = EN_3^2 = 4$ and $EN_4^2 = EN_5^2 = 9$. Find c such that

$$0.95 = P\left\{N_1^2 + N_2^2 + N_3^2 \leq c(N_4^2 + N_5^2)\right\}.$$

2. Let X_1, X_2 and X_3 be three independent normal random variables. X_1 is normal $N(0,4)$, X_2 is normal $N(0,12)$ and X_3 is normal $N(0,9)$. Compute

$$P\left\{\frac{X_1 + X_2}{|X_3|} \leq 3\right\}$$

using one of the enclosed tables.

3. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$f(t) = \begin{cases} 0, & \text{if } t < \theta \\ e^{-(t-\theta)}, & \text{if } t > \theta. \end{cases}$$

Find a moment estimator for θ, where θ is the unknown parameter.

4. Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with density function

$$h(t;\theta) = \begin{cases} 0, & \text{if } t \notin [1,3] \\ \theta 2^{-\theta}(t-1)^{\theta-1}, & \text{if } t \in [1,3] \end{cases}$$

Find a maximum liklihood estimator for θ.

5. Let X be a random variable with density function

$$h(t;\theta) = \frac{1}{2\theta}e^{-|t|/\theta}, \quad -\infty < t < \infty.$$

Compute the Cramer-Rao lower bound.

Here are some percentiles that may help you. If the percentile you need is not here, state which one you need and express your answer in terms of the unknown percentile. Note that $f_{.05}(3,2)$ is the fifth percentile of an f-distribution with 3 numerator and 2 denominator degrees of freedom. Similarly, $t_{.87}(1)$ is the 87th percentile of a t-distribution with 1 degree of freedom.

$f_{.05}(3,2) = .105$
$f_{.10}(3,2) = .183$
$f_{.90}(3,2) = 9.162$
$f_{.95}(3,2) = 19.164$
$t_{.87}(1) = 9/4$
$t_{.92}(2) = 9/4$
$t_{.95}(3) = 9/4$

Special Continuous Distributions

Notation and Parameters	Continuous pdf $f(x)$	Mean	Variance	MGF $M_X(t)$
Uniform $X \sim \mathrm{UNIF}(a, b)$, $a < b$	$\frac{1}{b-a}$, $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt} - e^{at}}{(b-a)t}$
Normal $X \sim \mathrm{N}(\mu, \sigma^2)$, $0 < \sigma^2$	$\frac{1}{\sqrt{2\pi}\,\sigma} e^{-[(x-\mu)/\sigma]^2/2}$	μ	σ^2	$e^{\mu t + \sigma^2 t^2/2}$
Gamma $X \sim \mathrm{GAM}(\theta, \kappa)$, $0 < \theta$, $0 < \kappa$	$\frac{1}{\theta^{\kappa}\Gamma(\kappa)} x^{\kappa-1} e^{-x/\theta}$, $0 < x$	$\kappa\theta$	$\kappa\theta^2$	$\left(\frac{1}{1-\theta t}\right)^{\kappa}$
Exponential $X \sim \mathrm{EXP}(\theta)$, $0 < \theta$	$\frac{1}{\theta} e^{-x/\theta}$, $0 < x$	θ	θ^2	$\frac{1}{1-\theta t}$
Two-Parameter Exponential $X \sim \mathrm{EXP}(\theta, \eta)$, $0 < \theta$	$\frac{1}{\theta} e^{-(x-\eta)/\theta}$, $\eta < x$	$\eta + \theta$	θ^2	$\frac{e^{\eta t}}{1-\theta t}$
Double Exponential $X \sim \mathrm{DE}(\theta, \eta)$, $0 < \theta$	$\frac{1}{2\theta} e^{-\lvert x-\eta\rvert/\theta}$	η	$2\theta^2$	$\frac{e^{\eta t}}{1-\theta^2 t^2}$

Special Continuous Distributions

Notation and Parameters	Continuous pdf $f(x)$	Mean	Variance	MGF $M_X(t)$
Weibull				
$X \sim \text{WEI}(\theta, \beta)$	$\frac{\beta}{\theta^\beta} x^{\beta-1} e^{-(x/\theta)^\beta}$	$\theta\Gamma\left(1+\frac{1}{\beta}\right)$	$\theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]$	*
$0<\theta$ $0<\beta$	$0<x$			
Extreme Value				
$X \sim \text{EV}(\theta, \eta)$	$\frac{1}{\theta}\exp\{[(x-\eta)/\theta]-\exp[(x-\eta)/\theta]\}$	$\eta-\gamma\theta$	$\frac{\pi^2\theta^2}{6}$	$e^{\eta t}\Gamma(1+\theta t)$
$0<\theta$		$\gamma \doteq 0.5772$ (Euler's const.)		
Cauchy				
$X \sim \text{CAU}(\theta, \eta)$	$\frac{1}{\theta\pi\{1+[(x-\eta)/\theta]^2\}}$	**	**	**
$0<\theta$				
Pareto				
$X \sim \text{PAR}(\theta, \kappa)$	$\frac{\kappa}{\theta(1+x/\theta)^{\kappa+1}}$	$\frac{\theta}{\kappa-1}$	$\frac{\theta^2\kappa}{(\kappa-2)(\kappa-1)^2}$	**
$0<\theta$ $0<\kappa$	$0<x$	$1<\kappa$	$2<\kappa$	
Chi-Square				
$X \sim \chi^2(\nu)$	$\frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2}$	ν	2ν	$\left(\frac{1}{1-2t}\right)^{\nu/2}$
$\nu=1,2,\ldots$	$0<x$			

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Special Continuous Distributions

Notation and Parameters	Continuous pdf $f(x)$	Mean	Variance	MGF $M_X(t)$
Student's t				
$X \sim t(\nu)$	$\dfrac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \dfrac{1}{\sqrt{\nu\pi}} \left(1+\dfrac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$	0	$\dfrac{\nu}{\nu-2}$	**
$\nu = 1, 2, \ldots$		$1 < \nu$	$2 < \nu$	
Snedecor's F				
$X \sim F(\nu_1, \nu_2)$	$\dfrac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\dfrac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}} x^{\frac{\nu_1}{2}-1}$	$\dfrac{\nu_2}{\nu_2-2}$	$\dfrac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$	**
$\nu_1 = 1, 2, \ldots$ $\nu_2 = 1, 2, \ldots$	$\times \left(1+\dfrac{\nu_1}{\nu_2}x\right)^{-\frac{\nu_1+\nu_2}{2}}$	$2 < \nu_2$	$4 < \nu_2$	
Beta				
$X \sim \mathrm{BETA}(a,b)$	$\dfrac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$	$\dfrac{a}{a+b}$	$\dfrac{ab}{(a+b+1)(a+b)^2}$	*
$0 < a$ $0 < b$	$0 < x < 1$			

*Not tractable.
**Does not exist.

Special Discrete Distributions

Notation and Parameters	Discrete pdf $f(x)$	Mean	Variance	MGF $M_X(t)$
Binomial				
$X \sim \text{BIN}(n, p)$ $0 < p < 1$ $q = 1 - p$	$\binom{n}{x} p^x q^{n-x}$ $x = 0, 1, \ldots, n$	np	npq	$(pe^t + q)^n$
Bernoulli				
$X \sim \text{BIN}(1, p)$ $0 < p < 1$ $q = 1 - p$	$p^x q^{1-x}$ $x = 0, 1$	p	pq	$pe^t + q$
Negative Binomial				
$X \sim \text{NB}(r, p)$ $0 < p < 1$ $r = 1, 2, \ldots$	$\binom{x-1}{r-1} p^r q^{x-r}$ $x = r, r+1, \ldots$	r/p	rq/p^2	$\left(\frac{pe^t}{1 - qe^t}\right)^r$
Geometric				
$X \sim \text{GEO}(p)$ $0 < p < 1$ $q = 1 - p$	pq^{x-1} $x = 1, 2, \ldots$	$1/p$	q/p^2	$\frac{pe^t}{1 - qe^t}$
Hypergeometric				
$X \sim \text{HYP}(n, M, N)$ $n = 1, 2, \ldots, N$ $M = 0, 1, \ldots, N$	$\binom{M}{x}\binom{N-M}{n-x} \Big/ \binom{N}{n}$ $x = 0, 1, \ldots, n$	nM/N	$n\frac{M}{N}\left(1 - \frac{M}{N}\right)\frac{N-n}{N-1}$	*
Poisson				
$X \sim \text{POI}(\mu)$ $0 < \mu$	$\frac{e^{-\mu}\mu^x}{x!}$ $x = 0, 1, \ldots$	μ	μ	$e^{\mu(e^t - 1)}$
Discrete Uniform				
$X \sim \text{DU}(N)$ $N = 1, 2, \ldots$	$1/N$ $x = 1, 2, \ldots, N$	$\frac{N+1}{2}$	$\frac{N^2 - 1}{12}$	$\frac{1}{N}\frac{e^t - e^{(N+1)t}}{1 - e^t}$

*Not tractable.