1. Let X_1 and X_2 be independent identically distributed random variables with density function

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Compute the density function of (Y_1, Y_2), where $Y_1 = X_1$ and $Y_2 = X_1 X_2$.

2. Let N be a standard normal random variable. Compute the moment generating function of $|N|$. (The moment generating function can be written with the help of the standard normal distribution function Φ.)

3. Let X_1 and X_2 be independent random variables with density functions

$$f(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

and

$$g(x) = \begin{cases} \frac{1}{2}e^{-(x-1)/2} & \text{if } x \geq 1 \\ 0 & \text{if } x < 1, \end{cases}$$

respectively. Compute the density function of $X_1 + X_2$.

4. Let X_1, X_2, $\ldots$, X_n be independent identically distributed random variables with cumulative distribution function

$$F(x) = \begin{cases} 1 - 1/x & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

Find the limiting distribution of $X_{1:n}^n$.

5. Let X_1, X_2, …, X_n be independent identically distributed random variables with density function

$$f(x) = \begin{cases} 1 & \text{if } x \in (0,1) \\ 0 & \text{if } x \notin (0,1). \end{cases}$$

Approximate $P\left(\sum_{i=1}^{90} X_i \leq 77\right)$ in terms of $\Phi(\cdot)$, the cdf of a standard normal.

1. (extra credit - 1 point) If X is exponentially distributed with parameter 1, what transformation of X will yield a uniform(0,1) random variable?

2. (extra credit - 1 point) If X is exponentially distributed with parameter 1, what transformation of X will yield a uniform(-1,1) random variable?

3. (extra credit - 1 point) Does congergence in probability imply convergence in distribution? If no, provide a counter example.

4. (extra credit - 1 point) Does convergence in distribution imply convergence in probability? If no, provide a counter example.

5. (extra credit - 1 point) State the law of large numbers including assumptions.

6. (extra credit - 1 point) State the central limit theorem including assumptions.