5080–Quiz

Name:

1. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with cumulative distribution function, $F(x) = \left(1 - \frac{1}{x^4}\right) 1\{x > 1\}$. Find the limiting distribution of $\frac{X_{n:n}}{n^{1/4}}$.

$$F_{\frac{X_{n:n}}{n^{1/4}}}(y) = \mathbb{P}\left(\frac{X_{n:n}}{n^{1/4}} \leq y\right) = \mathbb{P}\left(X_{n:n} \leq n^{1/4} y\right) = 1\{y > 0\}\left(1 - \frac{1}{n y^4}\right)^n$$

$$\longrightarrow \begin{cases} e^{-\frac{1}{y^4}} & y > 0, \\ 0 & o/w. \end{cases}$$

Note that $E(X)=\frac{1}{2}$, $E(X^2)=\int_0^1 x^2\,dx=\frac{1}{3}$, $var(X)=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$

2. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with density function, $f(x) = 1\{x \in (0,1)\}$. Approximate $P\left(\sum_{i=1}^{20} X_i \leq 12\right)$ in terms of $\Phi(\cdot)$, the cdf of a standard normal.

$$\mathbb{P}\left(\sum_{i=1}^{20} X_i \leq 12\right) = \mathbb{P}\left(\frac{\sum_{i=1}^{20} X_i - 10}{\sqrt{\frac{20}{12}}} \leq \frac{12-10}{\sqrt{\frac{20}{12}}}\right)$$

$$\approx \Phi\left(\frac{2}{\sqrt{\frac{20}{12}}}\right)$$

$$= \Phi\left(\frac{2}{\sqrt{\frac{5}{3}}}\right)$$