

5080-Quiz

There is a problem on the back.

Name:

1. Let X_1 and X_2 be independent identically distributed random variables with density functions

$$f_1(t) = f_2(t) = \begin{cases} 0, & \text{if } -\infty < t < 0 \\ e^{-t}, & \text{if } 0 \leq t < \infty. \end{cases}$$

Compute the density function of (Y_1, Y_2) , where $Y_1 = X_1$ and $Y_2 = X_1 + 2X_2$.

$$x_1 = y_1$$

$$x_2 = \left(\frac{y_2 - y_1}{2} \right)$$

$$J = \begin{bmatrix} 1 & 0 \\ -1/2 & 1/2 \end{bmatrix}$$

$$f_{Y_1, Y_2}(y_1, y_2) = \begin{cases} \frac{1}{2} e^{-\frac{1}{2}(y_1 + y_2)} & y_2 > y_1 > 0, \\ 0 & \text{o/w.} \end{cases}$$

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2. Let X be a random variable with density function

$$f(t) = \begin{cases} 0, & \text{if } t \notin [1, 3) \\ \frac{1}{2}, & \text{if } t \in [1, 3). \end{cases}$$

Compute the density function of $Y = (X - 1)^2$.

$$y = (x - 1)^2$$

$$\Rightarrow x - 1 = \sqrt{y} \quad \text{because } x - 1 > 0$$

$$\Rightarrow x = \sqrt{y} + 1$$

$$f_Y(y) = f_X(\sqrt{y} + 1) \frac{1}{2} y^{-\frac{1}{2}} \mathbb{1}_{\{y \in (0, 4)\}}$$

$$= \begin{cases} \frac{1}{4\sqrt{y}} & y \in (0, 4), \\ 0 & \text{o/w.} \end{cases}$$