

5080-Quiz

There is a problem on the back.

Name:

1. Let X be a random variable with density function

$$f(t) = \begin{cases} \frac{1}{4}, & \text{if } -2 \leq t \leq 2 \\ 0, & \text{if } t \notin [-2, 2] \end{cases}$$

Compute the density function of $Y = |X - 1|$.

$$F_Y(y) = P(|X-1| \leq y) = \begin{cases} P(-y \leq X-1 \leq y) & y \in (0, 3), \\ 0 & y \leq 0, \\ 1 & y \geq 3. \end{cases}$$

$$= \begin{cases} P(1-y \leq X \leq 1+y) \\ 0 \\ 1 \end{cases}$$

$$= \begin{cases} F_X(1+y) - F_X(1-y) \\ 0 \\ 1 \end{cases}$$

$$\Rightarrow f_Y(y) = \begin{cases} f_X(1+y) + f_X(1-y) & y \in (0, 3), \\ 0 & \text{o/w.} \end{cases} = \begin{cases} \frac{1}{2} & y \in (0, 1), \\ \frac{1}{4} & y \in (1, 3), \\ 0 & \text{o/w.} \end{cases}$$

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2. Let X be a random variable with density function

$$f(t) = \begin{cases} 0, & \text{if } t \notin [-1, 2] \\ \frac{1}{3}, & \text{if } t \in [-1, 2]. \end{cases}$$

Compute the density function of $Y = X^2$.

$$F_Y(y) = P(X^2 \leq y) = \begin{cases} P(-\sqrt{y} \leq X \leq \sqrt{y}) & y \in (0, 4) \\ 0 & y \leq 0 \\ 1 & y \geq 4 \end{cases}$$

$$= \begin{cases} F_X(\sqrt{y}) - F_X(-\sqrt{y}) & y \in (0, 4) \\ 0 & y \leq 0 \\ 1 & y \geq 4 \end{cases}$$

$$\Rightarrow f_Y(y) = \begin{cases} f_X(\sqrt{y}) \frac{1}{2} y^{-\frac{1}{2}} + f_X(-\sqrt{y}) \frac{1}{2} y^{-\frac{1}{2}} & y \in (0, 4), \\ 0 & \text{o/w.} \end{cases}$$

$$= \begin{cases} \frac{1}{3\sqrt{y}} & y \in (0, 1), \\ \frac{1}{6\sqrt{y}} & y \in (1, 4), \\ 0 & \text{o/w.} \end{cases}$$