

5080-Quiz

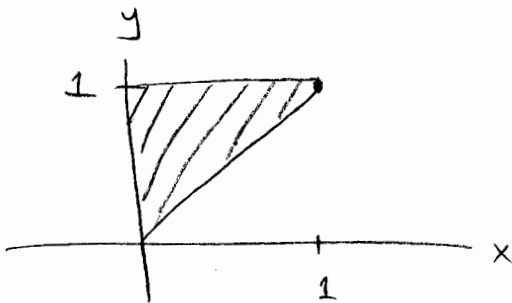
There is a problem on the back.

Name:

1. Let

$$f(x, y) = k(x + y), \quad 0 \leq x \leq y \leq 1.$$

Find k such that $f(x, y)$ is a bivariate density function.



$$\begin{aligned}
 1 &= \int_0^1 \int_x^1 k(x+y) \, dy \, dx = k \int_0^1 \left[xy + \frac{y^2}{2} \right] \Big|_{y=x}^{y=1} \, dx \\
 &= k \int_0^1 \left(x + \frac{1}{2} \right) - \left(x^2 + \frac{x^2}{2} \right) \, dx \\
 &= k \int_0^1 \left(-\frac{3}{2}x^2 + x + \frac{1}{2} \right) \, dx \\
 &= k \left[-\frac{1}{2}x^3 + \frac{x^2}{2} + \frac{x}{2} \right] \Big|_0^1 \\
 &= k \left[-\frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right] \\
 &= k/2.
 \end{aligned}$$

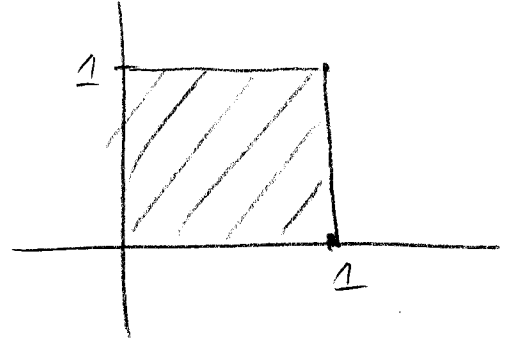
Therefore, $k=2$.

2. Let

$$f(x, y) = \frac{2}{3}(x+1), \quad 0 \leq x \leq 1, 0 \leq y \leq 1$$

denote the joint density of (X, Y) .

- (a) Compute the density of X .
- (b) Compute the density of Y .



$$f(x, y) = \mathbb{1}\{y \in (0, 1)\} \cdot \frac{2}{3}(x+1) \mathbb{1}\{x \in (0, 1)\}$$

$$\Rightarrow f_X(x) = \frac{2}{3}(x+1) \mathbb{1}\{x \in (0, 1)\}$$

$$\text{and } f_Y(y) = \mathbb{1}\{y \in (0, 1)\}.$$